

You must **SHOW WORK** for full credit. Box your answers. *No Calculators Allowed.*

Show all work for credit. Box your answer if appropriate. YOU HAVE 80 MINUTES FOR THIS TEST.

1. Establish the identity using either method.

$$\tan \theta \cot \theta - \cos^2 \theta = \sin^2 \theta$$

$$\frac{\sin \theta / \cos \theta}{\cos \theta (\sin \theta)} - \cos^2 \theta = \sin^2 \theta$$

$$1 - \cos^2 \theta = \sin^2 \theta$$

$$\sin^2 \theta = \sin^2 \theta \checkmark$$

2. Establish the identity using either method.

$$\cos \theta (\tan \theta + \cot \theta) = \csc \theta$$

$$\cos \theta \tan \theta + \cos \theta \cot \theta = \csc \theta$$

$$\cos \theta \left(\frac{\sin \theta}{\cos \theta} \right) + \cos \theta \left(\frac{\cos \theta}{\sin \theta} \right) = \csc \theta$$

$$\sin \theta + \frac{\cos^2 \theta}{\sin \theta} = \csc \theta$$

$$\frac{\sin^2 \theta}{\sin \theta} + \frac{\cos^2 \theta}{\sin \theta} = \csc \theta$$

$$\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta} = \csc \theta$$

$$\frac{1}{\sin \theta} = \csc \theta$$

$$\csc \theta = \csc \theta \checkmark$$

3. Find the exact value of:

$$\sin 140^\circ \cos 10^\circ + \sin 10^\circ \cos 140^\circ$$

$$\sin(\alpha + \beta) = \sin(140^\circ + 10^\circ)$$

$$\sin 150^\circ = \frac{1}{2}$$

4. Find the exact value of (& draw pictures):

$$\cos(\alpha - \beta)$$

$$\text{if } \cos \alpha = \frac{-3}{5}; \frac{\pi}{2} < \alpha < \pi \text{ and}$$

$$\sin \beta = \frac{-5}{13}; \frac{3\pi}{2} < \beta < 2\pi$$

$$\cos \alpha = \frac{-3}{5} \quad x = -3 \quad \sin \alpha = \frac{4}{5}$$

$$\sin \beta = \frac{-5}{13} \quad y = -5 \quad \cos \beta = \frac{12}{13}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$= \left(\frac{-3}{5} \right) \left(\frac{12}{13} \right) + \left(\frac{4}{5} \right) \left(\frac{-5}{13} \right)$$

$$= \frac{-36}{65} - \frac{20}{65} = \frac{-56}{65}$$

5. Use Half Angle Formula to find the exact value

of: $\sin\left(\frac{\pi}{8}\right)$ (reduce/simplify answer!)

$$\sin\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 - \cos\alpha}{2}}$$

$$\frac{\alpha}{2} = \frac{\pi}{8} \Rightarrow \alpha = \frac{2\pi}{8} = \frac{\pi}{4}$$

$$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$0 \leq \alpha \leq \frac{\pi}{2}$$

$$\sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2 - \sqrt{2}}{4}}$$

$$\frac{\sqrt{2 - \sqrt{2}}}{2}$$

6. Use Sum/Difference Formula to find the exact

value of: $\sin 195^\circ$ (reduce/simplify answer!)

$$\begin{aligned} \sin 195^\circ &= \sin(150^\circ + 45^\circ) \\ &= \sin 150^\circ \cos 45^\circ + \cos 150^\circ \sin 45^\circ \\ &= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \end{aligned}$$

$$\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$\frac{\sqrt{2} - \sqrt{6}}{4}$$

7. Find the exact value of: $\sec\left(\tan^{-1}\frac{-5}{12}\right)$

Draw picture!

$$\theta = \tan^{-1}\left(\frac{-5}{12}\right)$$

$$\tan\theta = \frac{-5}{12} = \frac{y}{x}$$

$$\sec\theta = \frac{r}{x} = \frac{13}{12}$$

$$\theta = \sec^{-1}\left(\frac{13}{12}\right)$$

$$\sec\left(\sec^{-1}\left(\frac{13}{12}\right)\right) = \frac{13}{12}$$

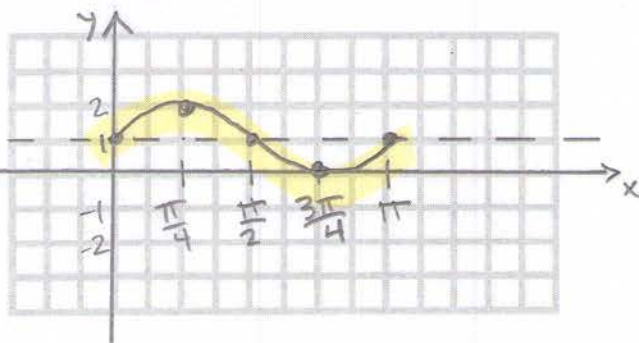
$$\begin{aligned} y &= -5 \\ x &= 12 \\ r &= 13 \end{aligned}$$

8. Graph and give the five critical values within a

single period.

$$y = \sin(2\theta) + 1$$

$$P = \frac{2\pi}{2} = \pi$$



amp: 1 per: π vert: 1 horiz: N/A

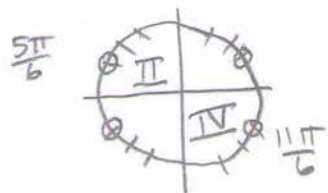
Critical values: $(0, 1)$, $(\frac{\pi}{4}, 2)$, $(\frac{\pi}{2}, 1)$, $(\frac{3\pi}{4}, 0)$, $(\pi, 1)$

9. Solve the equation on the interval $0 \leq \theta < 2\pi$.

$$\tan \theta = -\frac{\sqrt{3}}{3}$$

$$\frac{1}{2} = \sin \theta$$

$$\frac{\sqrt{3}}{2} = \cos \theta$$



$$\theta = \frac{5\pi}{6} + \pi k$$

$$\theta = \frac{5\pi}{6}, \frac{11\pi}{6}$$

10. Solve the equation on the interval $0 \leq \theta < 2\pi$.

$$\sin 3\theta = -1 \quad \alpha = 3\theta$$

$$\sin \alpha = -1$$

$$\alpha = \frac{3\pi}{2} + 2\pi k$$

$$3\theta = \frac{3\pi}{2} + 2\pi k$$

$$\theta = \frac{\pi}{2} + \frac{2\pi}{3} k$$

$$\theta = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

11. Find the exact value: draw pictures!

$$\cos \left[\sin^{-1} \left(\frac{1}{2} \right) - \tan^{-1} \left(-\frac{4}{3} \right) \right]$$

$$\cos [\alpha - \beta]$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin^{-1} \left(\frac{1}{2} \right) = \alpha \quad \sin \alpha = \frac{1}{2} \quad \cos \alpha = \frac{\sqrt{3}}{2}$$

$$\tan^{-1} \left(-\frac{4}{3} \right) = \beta \quad \tan \beta = -\frac{4}{3} \quad \begin{array}{l} y = -4 \\ x = 3 \\ r = 5 \end{array}$$

$$\cos \beta = \frac{3}{5} \quad \sin \beta = -\frac{4}{5}$$

$$\cos(\alpha - \beta) = \left(\frac{\sqrt{3}}{2} \right) \left(\frac{3}{5} \right) + \left(\frac{1}{2} \right) \left(-\frac{4}{5} \right)$$

$$= \frac{3\sqrt{3}}{10} - \frac{4}{10}$$

$$\frac{3\sqrt{3} - 4}{10}$$

12. Solve the inequality. Express your answer in Set Notation. Show test points.

$$x^2 + 2x - 10 < 25$$

$$x^2 + 2x - 35 < 0$$

$$(x - 5)(x + 7) < 0$$

$$\text{C.V. } 5, -7$$



$$f(0) = -10 < 25 \text{ True}$$

\therefore

$$-7 < x < 5$$