

Measuring and Constructing Segments

CC.9-12.G.CO.12 Make formal geometric constructions with a variety of tools and methods...

Objectives

Use length and midpoint of a segment.

Construct midpoints and congruent segments.

Vocabulary

coordinate
distance
length
congruent segments
construction
between
midpoint
bisect
segment bisector

Why learn this?

You can measure a segment to calculate the distance between two locations. Maps of a race are used to show the distance between stations on the course. (See Example 4.)

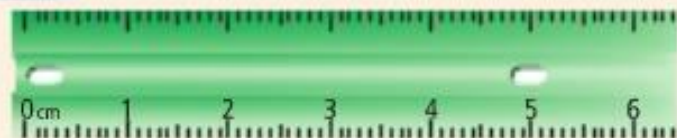
A ruler can be used to measure the distance between two points. A point corresponds to one and only one number on the ruler. This number is called a **coordinate**. The following postulate summarizes this concept.



Postulate 1-2-1

Ruler Postulate

The points on a line can be put into a one-to-one correspondence with the real numbers.



Know it!

Note

The **distance** between any two points is the absolute value of the difference of the coordinates. If the coordinates of points A and B are a and b , then the distance between A and B is $|a - b|$ or $|b - a|$. The distance between A and B is also called the **length** of \overline{AB} , or AB .



EXAMPLE 1 Finding the Length of a Segment

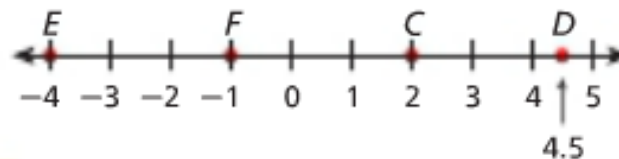
Find each length.

A DC

$$\begin{aligned} DC &= |4.5 - 2| \\ &= |2.5| \\ &= 2.5 \end{aligned}$$

B EF

$$\begin{aligned} EF &= |-4 - (-1)| \\ &= |-4 + 1| \\ &= |-3| \\ &= 3 \end{aligned}$$



Caution!

PQ represents a number, while \overline{PQ} represents a geometric figure. Be sure to use equality for numbers ($PQ = RS$) and congruence for figures ($\overline{PQ} \cong \overline{RS}$).

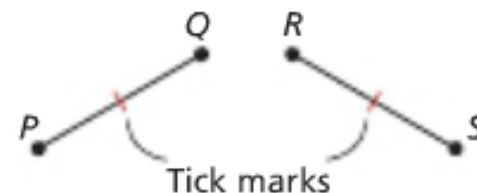


Find each length.

- XY
- XZ



Congruent segments are segments that have the same length. In the diagram, $PQ = RS$, so you can write $\overline{PQ} \cong \overline{RS}$. This is read as “segment PQ is congruent to segment RS .” **Tick marks** are used in a figure to show congruent segments.



You can make a sketch or measure and draw a segment. These may not be exact. A **construction** is a way of creating a figure that is more precise. One way to make a geometric construction is to use a compass and straightedge.

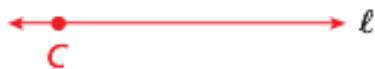


Construction Congruent Segment

Construct a segment congruent to \overline{AB} .

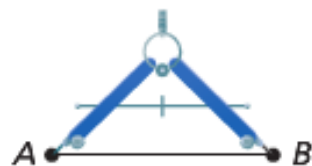


1



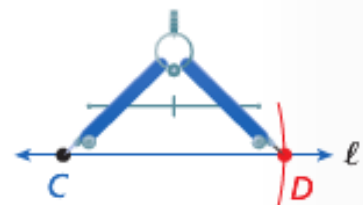
Draw ℓ . Choose a point on ℓ and label it C .

2



Open the compass to distance AB .

3



Place the point of the compass at C and make an arc through ℓ . Find the point where the arc and ℓ intersect and label it D .

$$\overline{CD} \cong \overline{AB}$$

EXAMPLE**2****Copying a Segment**

Sketch, draw, and construct a segment congruent to \overline{MN} .

Step 1 Estimate and sketch.

Estimate the length of \overline{MN} and sketch \overline{PQ} approximately the same length.

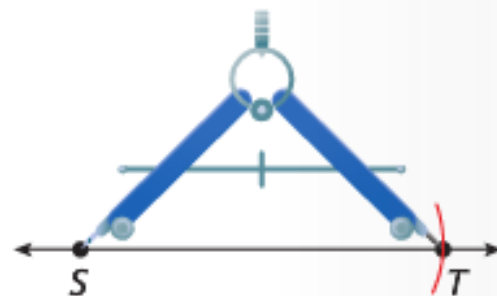
Step 2 Measure and draw.

Use a ruler to measure \overline{MN} . MN appears to be 3.1 cm. Use a ruler and draw \overline{XY} to have length 3.1 cm.

Step 3 Construct and compare.

Use a compass and straightedge to construct \overline{ST} congruent to \overline{MN} .

A ruler shows that \overline{PQ} and \overline{XY} are approximately the same length as \overline{MN} , but \overline{ST} is precisely the same length.



2. Sketch, draw, and construct a segment congruent to \overline{JK} .



In order for you to say that a point B is **between** two points A and C , all three of the points must lie on the same line, and $AB + BC = AC$.



Postulate 1-2-2

Segment Addition Postulate

If B is between A and C ,
then $AB + BC = AC$.



EXAMPLE 3

3

Using the Segment Addition Postulate

x^2y Algebra

A B is between A and C , $AC = 14$, and $BC = 11.4$. Find AB .

$$AC = AB + BC$$

$$14 = AB + 11.4$$

$$\underline{- 11.4} \quad \underline{- 11.4}$$

$$2.6 = AB$$

Seg. Add. Post.

Substitute 14 for AC and 11.4 for BC .

Subtract 11.4 from both sides.

Simplify.

B S is between R and T . Find RT .

$$RT = RS + ST$$

$$4x = (2x + 7) + 28$$

$$4x = 2x + 35$$

$$\underline{- 2x} \quad \underline{- 2x}$$

$$2x = 35$$

$$\frac{2x}{2} = \frac{35}{2}$$

$$x = \frac{35}{2}, \text{ or } 17.5$$

$$RT = 4x$$

$$= 4(17.5) = 70$$

Seg. Add. Post.

Substitute the given values.

Simplify.

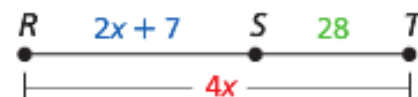
Subtract $2x$ from both sides.

Simplify.

Divide both sides by 2.

Simplify.

Substitute 17.5 for x .

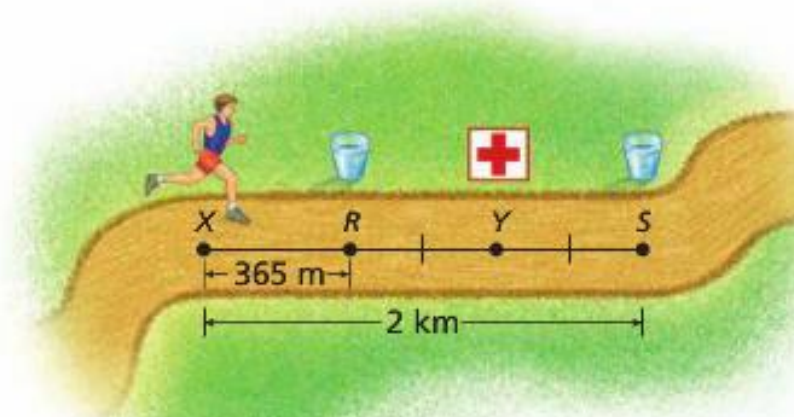


The **midpoint** M of \overline{AB} is the point that **bisects**, or divides, the segment into two congruent segments. If M is the midpoint of \overline{AB} , then $AM = MB$. So if $AB = 6$, then $AM = 3$ and $MB = 3$.

EXAMPLE 4 Recreation Application

x²y Algebra

The map shows the route for a race. You are 365 m from drink station R and 2 km from drink station S . The first-aid station is located at the midpoint of the two drink stations. How far are you from the first-aid station?



Let your current location be X and the location of the first-aid station be Y .

$$XR + RS = XS$$

$$365 + RS = 2000$$

$$\begin{array}{r} 365 \\ - 365 \\ \hline \end{array} \quad \begin{array}{r} RS \\ - 365 \\ \hline \end{array}$$

$$RS = 1635$$

$$RY = 817.5$$

$$XY = XR + RY$$

$$= 365 + 817.5 = 1182.5 \text{ m}$$

Seg. Add. Post.

Substitute 365 for XR and 2000 for XS .

Subtract 365 from both sides.

Simplify.

Y is the mdpt. of \overline{RS} , so $RY = \frac{1}{2}RS$.

Substitute 365 for XR and 817.5 for RY .

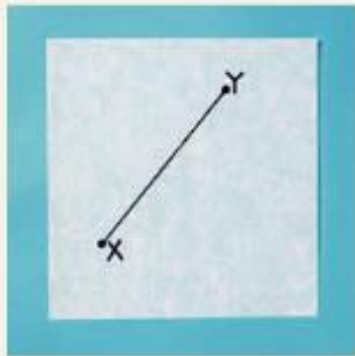
You are 1182.5 m from the first-aid station.

A **segment bisector** is any ray, segment, or line that intersects a segment at its midpoint. It divides the segment into two equal parts at its midpoint.



Construction Segment Bisector

1



Draw \overline{XY} on a sheet of paper.

2



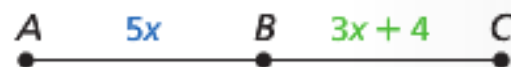
Fold the paper so that Y is on top of X .

3



Unfold the paper. The line represented by the crease bisects \overline{XY} . Label the midpoint M .

$$XM = MY$$

EXAMPLE**5****Using Midpoints to Find Lengths**

B is the midpoint of \overline{AC} , $AB = 5x$, and $BC = 3x + 4$. Find AB , BC , and AC .

 x^2y Algebra**Step 1** Solve for x .

$$AB = BC$$

$$5x = 3x + 4$$

$$\begin{array}{r} -3x \\ \hline \end{array} \quad \begin{array}{r} -3x \\ \hline \end{array}$$

$$2x = 4$$

$$\frac{2x}{2} = \frac{4}{2}$$

$$x = 2$$

 *B is the mdpt. of \overline{AC} .**Substitute $5x$ for AB and $3x + 4$ for BC .**Subtract $3x$ from both sides.**Simplify.**Divide both sides by 2.**Simplify.***Step 2** Find AB , BC , and AC .

$$AB = 5x$$

$$= 5(2) = 10$$

$$BC = 3x + 4$$

$$= 3(2) + 4 = 10$$

$$AC = AB + BC$$

$$= 10 + 10 = 20$$



5. S is the midpoint of \overline{RT} , $RS = -2x$, and $ST = -3x - 2$. Find RS , ST , and RT .

THINK AND DISCUSS

- Suppose R is the midpoint of \overline{ST} . Explain how SR and ST are related.
- GET ORGANIZED** Copy and complete the graphic organizer. Make a sketch and write an equation to describe each relationship.

	B is between A and C .	B is the midpoint of \overline{AC} .
Sketch		
Equation		

Know it!

Note