COMMON CORE

1-3 Measuring and Constructing Angles

CC.9-12.G.CO.12 Make formal geometric constructions with a variety of tools and methods… *Also* CC.9-12.G.CO.1

Objectives

Name and classify angles.

Measure and construct angles and angle bisectors.

Vocabulary

angle
vertex
interior of an angle
exterior of an angle
measure
degree
acute angle
right angle
obtuse angle
straight angle
congruent angles
angle bisector

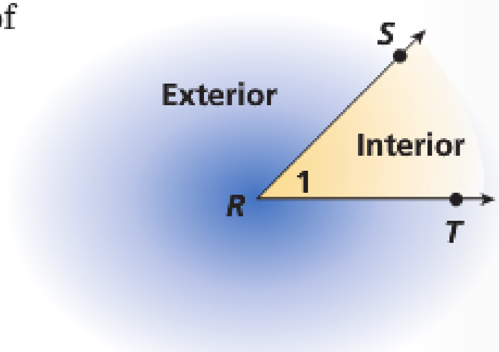
Who uses this?

Surveyors use angles to help them measure and map the earth's surface. (See Exercise 27.)

A transit is a tool for measuring angles. It consists of a telescope that swivels horizontally and vertically. Using a transit, a surveyor can measure the *angle* formed by his or her location and two distant points.

An **angle** is a figure formed by two rays, or sides, with a common endpoint called the **vertex** (plural: *vertices*). You can name an angle several ways: by its vertex, by a point on each ray and the vertex, or by a number.

The set of all points between the sides of the angle is the **interior of an angle**. The **exterior of an angle** is the set of all points outside the angle.



Angle Name

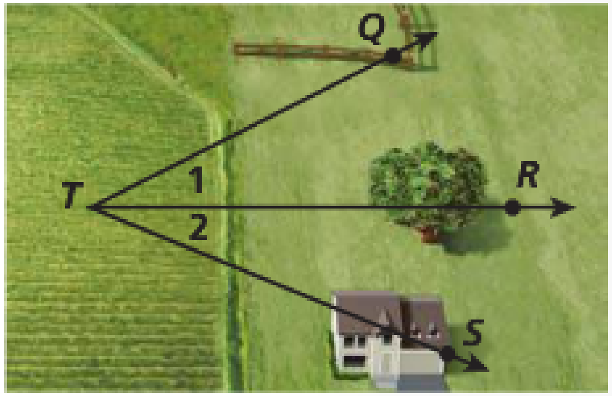
$\angle R$, $\angle SRT$, $\angle TRS$, or $\angle 1$

You cannot name an angle just by its vertex if the point is the vertex of more than one angle. In this case, you must use all three points to name the angle, and the middle point is always the vertex.

EXAMPLE 1

Naming Angles

A surveyor recorded the angles formed by a transit (point T) and three distant points, Q, R, and S. Name three of the angles.



$\angle QTR$, $\angle QTS$, and $\angle RTS$

CHECK IT OUT!

1. Write the different ways you can name the angles in the diagram.

The **measure** of an angle is usually given in degrees. Since there are 360° in a circle, one **degree** is $\frac{1}{360}$ of a circle. When you use a protractor to measure angles, you are applying the following postulate.

Know it! Note

Postulate 1-3-1 Protractor Postulate

Given $\overleftrightarrow{AB}$ and a point O on $\overleftrightarrow{AB}$, all rays that can be drawn from O can be put into a one-to-one correspondence with the real numbers from 0 to 180.

Student to Student

Using a Protractor

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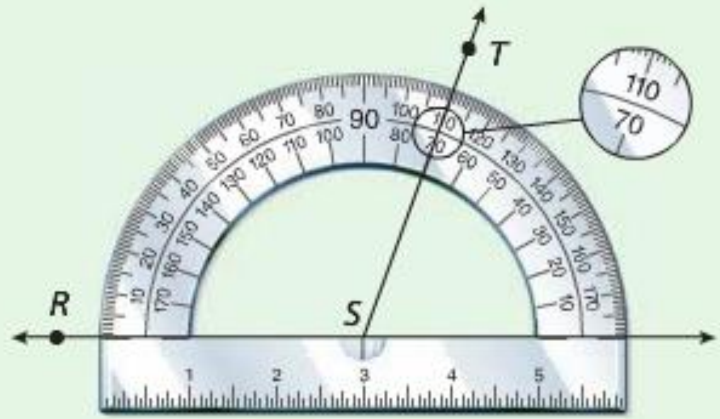
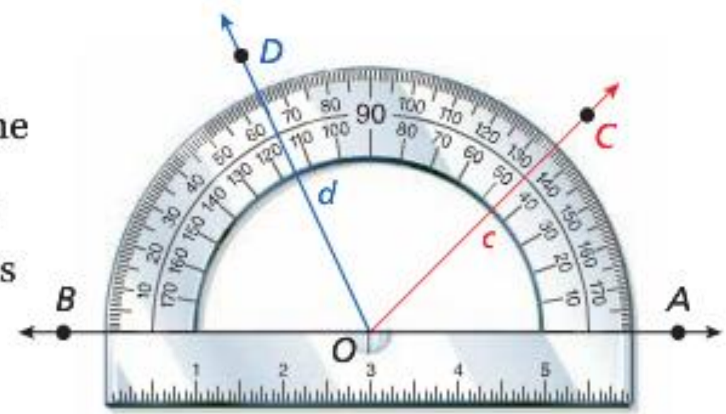
Most protractors have two sets of numbers around the edge. When I measure an angle and need to know which number to use, I first ask myself whether the angle is acute, right, or obtuse. For example, ∠RST looks like it is obtuse, so I know its measure must be 110°, not 70°.

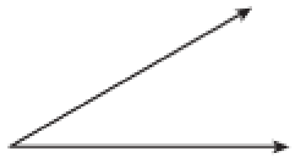
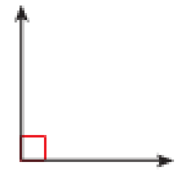
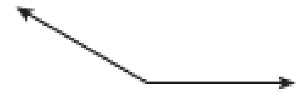


You can use the Protractor Postulate to help you classify angles by their measure. The measure of an angle is the absolute value of the difference of the real numbers that the rays correspond with on a protractor. If $\overrightarrow{OC}$ corresponds with c and $\overrightarrow{OD}$ corresponds with d, $m\angle DOC = |d - c|$ or $|c - d|$.



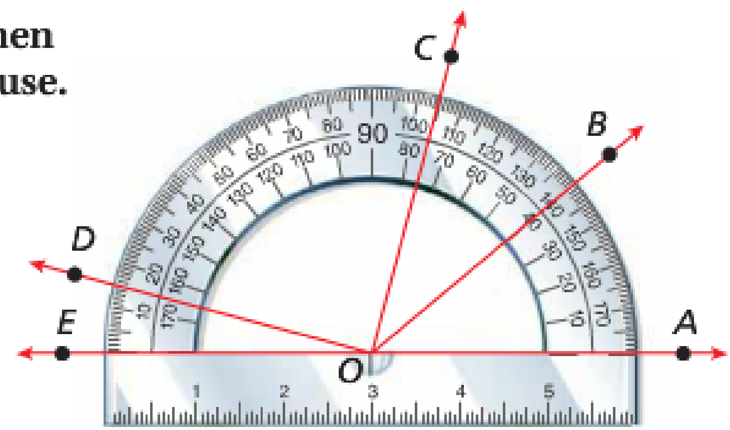
Know it! Note

Types of Angles

Acute Angle	Right Angle	Obtuse Angle	Straight Angle
			
Measures greater than 0° and less than 90°	Measures 90°	Measures greater than 90° and less than 180°	Formed by two opposite rays and meaures 180°

EXAMPLE 2 Measuring and Classifying Angles

Find the measure of each angle. Then classify each as acute, right, or obtuse.



A $\angle AOD$

$m\angle AOD = 165°$

$\angle AOD$ is obtuse.

B $\angle COD$

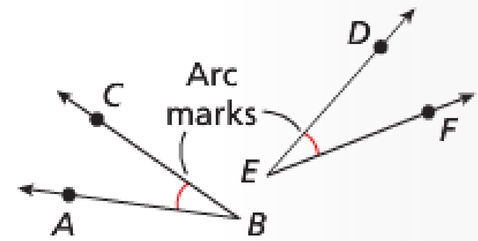
$m\angle COD = |165 - 75| = 90°$

$\angle COD$ is a right angle.

CHECK IT OUT! **Use the diagram to find the measure of each angle. Then classify each as acute, right, or obtuse.**

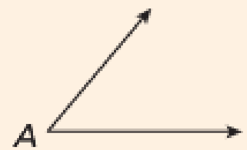
2a. $\angle BOA$ **2b.** $\angle DOB$ **2c.** $\angle EOC$

Congruent angles are angles that have the same measure. In the diagram, $m\angle ABC = m\angle DEF$, so you can write $\angle ABC \cong \angle DEF$. This is read as "angle *ABC* is congruent to angle *DEF*." ***Arc marks*** are used to show that the two angles are congruent.

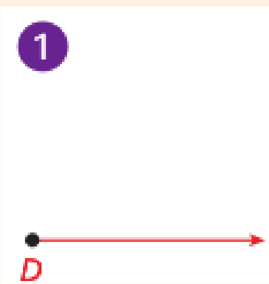


Construction Congruent Angle

Construct an angle congruent to ∠A.

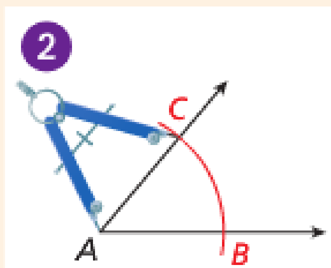


1



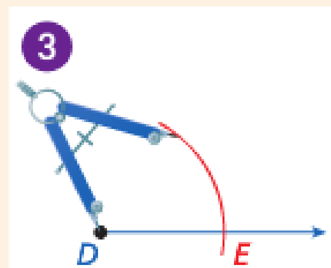
Use a straightedge to draw a ray with endpoint ***D***.

2



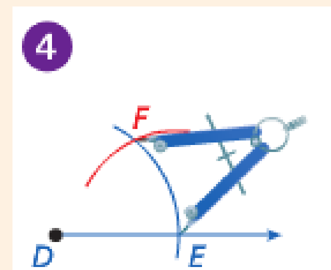
Place the compass point at *A* and draw an arc that intersects both sides of ∠*A*. Label the intersection points ***B*** and ***C***.

3



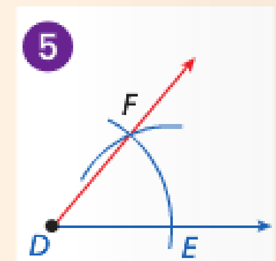
Using the same compass setting, place the compass point at ***D*** and draw an arc that intersects the ray. Label the intersection ***E***.

4



Place the compass point at *B* and open it to the distance *BC*. Place the point of the compass at ***E*** and draw an arc. Label its intersection with the first arc ***F***.

5



Use a straightedge to draw $\overrightarrow{DF}$.

$\angle D \cong \angle A$

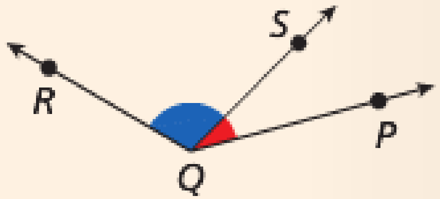
The Angle Addition Postulate is very similar to the Segment Addition Postulate that you learned in the previous lesson.

Know it! Note

Postulate 1-3-2 **Angle Addition Postulate**

If S is in the interior of $\angle PQR$, then $m\angle PQS + m\angle SQR = m\angle PQR$.

($\angle$ Add. Post.)

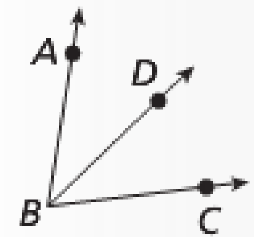


EXAMPLE 3

x^2y Algebra

Using the Angle Addition Postulate

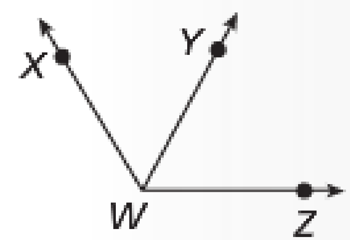
$m\angle ABD = 37°$ and $m\angle ABC = 84°$. Find $m\angle DBC$.



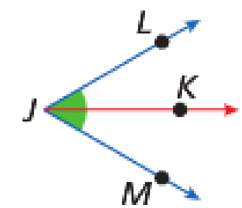
$m\angle ABC = m\angle ABD + m\angle DBC$	*$\angle$ Add. Post.*
$84° = 37° + m\angle DBC$	*Substitute the given values.*
$\underline{-37 \quad -37}$	*Subtract 37 from both sides.*
$47° = m\angle DBC$	*Simplify.*

CHECK IT OUT!

3. $m\angle XWZ = 121°$ and $m\angle XWY = 59°$. Find $m\angle YWZ$.

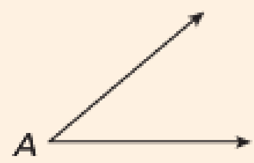


An **angle bisector** is a ray that divides an angle into two congruent angles. $\overrightarrow{JK}$ bisects $\angle LJM$; thus $\angle LJK \cong \angle KJM$.

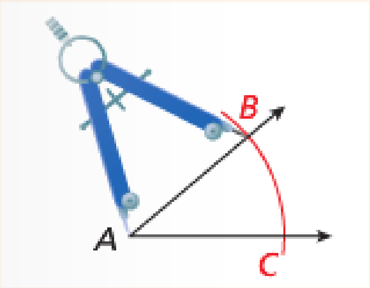


Construction Angle Bisector

Construct the bisector of ∠A.

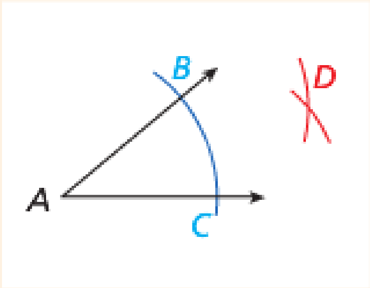


1



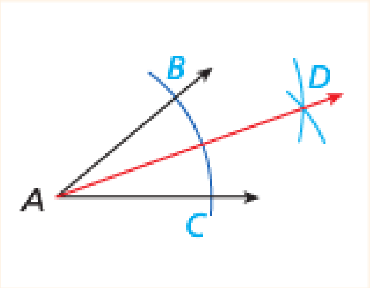
Place the point of the compass at *A* and draw an arc. Label its points of intersection with ∠*A* as ***B*** and ***C***.

2



Without changing the compass setting, draw intersecting arcs from *B* and *C*. Label the intersection of the arcs as ***D***.

3



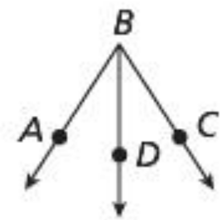
Use a straightedge to draw $\overrightarrow{AD}$.

$\overrightarrow{AD}$ bisects ∠*A*.

EXAMPLE 4 Finding the Measure of an Angle

Algebra

$\overrightarrow{BD}$ bisects $\angle ABC$, $m\angle ABD = (6x+3)^\circ$, and $m\angle DBC = (8x-7)^\circ$. Find $m\angle ABD$.



Step 1 Find x.

$m\angle ABD = m\angle DBC$	*Def. of ∠ bisector*
$(6x+3)^\circ = (8x-7)^\circ$	*Substitute the given values.*
$\underline{+7} \quad \underline{+7}$	*Add 7 to both sides.*
$6x + 10 = 8x$	*Simplify.*
$\underline{-6x} \quad \underline{-6x}$	*Subtract 6x from both sides.*
$10 = 2x$	*Simplify.*
$\frac{10}{2} = \frac{2x}{2}$	*Divide both sides by 2.*
$5 = x$	*Simplify.*

Step 2 Find $m\angle ABD$.

$m\angle ABD = 6x + 3$	
$= 6(5) + 3$	*Substitute 5 for x.*
$= 33^\circ$	*Simplify.*

CHECK IT OUT! **Find the measure of each angle.**

4a. $\overrightarrow{QS}$ bisects $\angle PQR$, $m\angle PQS = (5y-1)^\circ$, and $m\angle PQR = (8y+12)^\circ$. Find $m\angle PQS$.

4b. $\overrightarrow{JK}$ bisects $\angle LJM$, $m\angle LJK = (-10x+3)^\circ$, and $m\angle KJM = (-x+21)^\circ$. Find $m\angle LJM$.

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Explain why any two right angles are congruent.

2. $\overrightarrow{BD}$ bisects $\angle ABC$. How are $m\angle ABC$, $m\angle ABD$, and $m\angle DBC$ related?

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In the cells sketch, measure, and name an example of each angle type.

	Diagram	Measure	Name
Acute Angle			
Right Angle			
Obtuse Angle			
Straight Angle			