COMMON CORE

1-6 Midpoint and Distance in the Coordinate Plane

CC.9-12.G.GPE.7 Use coordinates to compute perimeters…, e.g., using the distance formula.*

Objectives

Develop and apply the formula for midpoint.

Use the Distance Formula and the Pythagorean Theorem to find the distance between two points.

Vocabulary

coordinate plane
leg
hypotenuse

Why learn this?

You can use a coordinate plane to help you calculate distances. (See Example 5.)

Major League baseball fields are laid out according to strict guidelines. Once you know the dimensions of a field, you can use a coordinate plane to find the distance between two of the bases.

A **coordinate plane** is a plane that is divided into four regions by a horizontal line (x-axis) and a vertical line (y-axis). The location, or coordinates, of a point are given by an ordered pair (x, y).

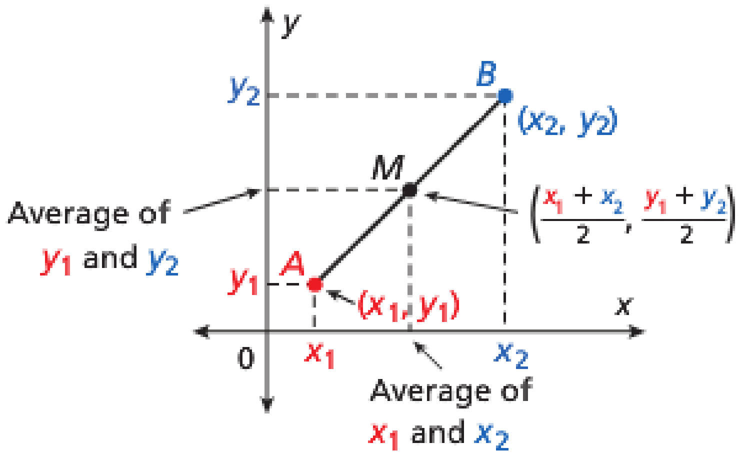
You can find the midpoint of a segment by using the coordinates of its endpoints. Calculate the average of the x-coordinates and the average of the y-coordinates of the endpoints.

Know it! Note

Midpoint Formula

The midpoint M of $\overline{AB}$ with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ is found by

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$



EXAMPLE 1

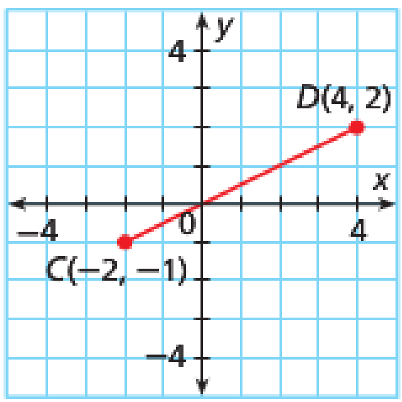
Finding the Coordinates of a Midpoint

Find the coordinates of the midpoint of $\overline{CD}$ with endpoints $C(-2, -1)$ and $D(4, 2)$.

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\frac{-2+4}{2}, \frac{-1+2}{2} = \left(\frac{2}{2}, \frac{1}{2}\right)$$

$$= \left(1, \frac{1}{2}\right)$$



Helpful Hint

To make it easier to picture the problem, plot the segment's endpoints on a coordinate plane.

CHECK IT OUT!

1. Find the coordinates of the midpoint of $\overline{EF}$ with endpoints $E(-2, 3)$ and $F(5, -3)$.

EXAMPLE 2 Finding the Coordinates of an Endpoint

Algebra

M is the midpoint of $\overline{AB}$. A has coordinates $(2, 2)$, and M has coordinates $(4, -3)$. Find the coordinates of B.

Step 1 Let the coordinates of B equal (x, y).

Step 2 Use the Midpoint Formula: $(4, -3) = \left(\frac{2 + x}{2}, \frac{2 + y}{2}\right)$.

Step 3 Find the x-coordinate. Find the y-coordinate.

Find the x-coordinate.		Find the y-coordinate.
$4 = \frac{2 + x}{2}$	*Set the coordinates equal.*	$-3 = \frac{2 + y}{2}$
$2(4) = 2\left(\frac{2 + x}{2}\right)$	*Multiply both sides by 2.*	$2(-3) = 2\left(\frac{2 + y}{2}\right)$
$8 = 2 + x$	*Simplify.*	$-6 = 2 + y$
$-2 \quad -2$	*Subtract 2 from both sides.*	$-2 \quad -2$
$6 = x$	*Simplify.*	$-8 = y$

The coordinates of B are $(6, -8)$.

CHECK IT OUT!

2. S is the midpoint of $\overline{RT}$. R has coordinates $(-6, -1)$, and S has coordinates $(-1, 1)$. Find the coordinates of T.

The Ruler Postulate can be used to find the distance between two points on a number line. The Distance Formula is used to calculate the distance between two points in a coordinate plane.

Distance Formula

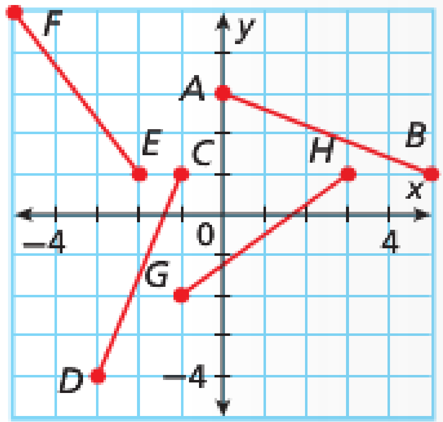
In a coordinate plane, the distance d between two points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

EXAMPLE 3

Using the Distance Formula

Find AB and CD. Then determine if $\overline{AB} \cong \overline{CD}$.



Step 1 Find the coordinates of each point.

$A(0, 3)$, $B(5, 1)$, $C(-1, 1)$, and $D(-3, -4)$

Step 2 Use the Distance Formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(5 - 0)^2 + (1 - 3)^2} = \sqrt{5^2 + (-2)^2} = \sqrt{25 + 4} = \sqrt{29}$$

$$CD = \sqrt{[-3 - (-1)]^2 + (-4 - 1)^2} = \sqrt{(-2)^2 + (-5)^2} = \sqrt{4 + 25} = \sqrt{29}$$

Since $AB = CD$, $\overline{AB} \cong \overline{CD}$.

CHECK IT OUT!

3. Find EF and GH. Then determine if $\overline{EF} \cong \overline{GH}$.

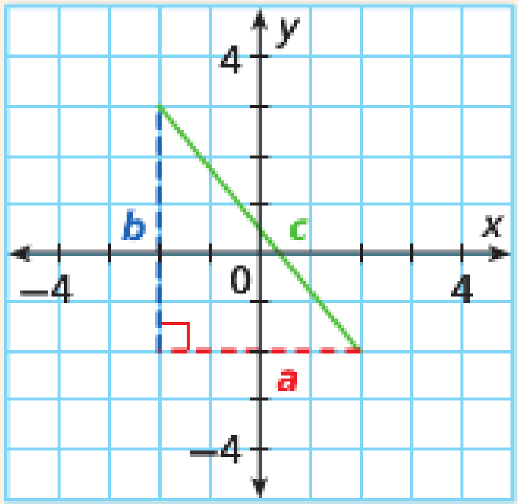
You can also use the Pythagorean Theorem to find the distance between two points in a coordinate plane. You will learn more about the Pythagorean Theorem later in this course.

In a right triangle, the two sides that form the right angle are the **legs**. The side across from the right angle that stretches from one leg to the other is the **hypotenuse**. In the diagram, a and b are the lengths of the shorter sides, or legs, of the right triangle. The longest side is called the hypotenuse and has length c.

Theorem 1-6-1 Pythagorean Theorem

In a right triangle, the sum of the squares of the lengths of the *legs* is equal to the square of the length of the *hypotenuse*.

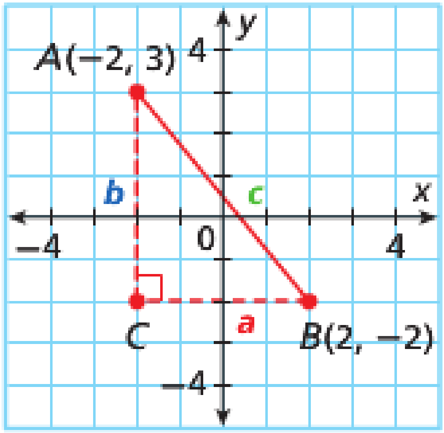
$$a^2 + b^2 = c^2$$



EXAMPLE 4

Finding Distances in the Coordinate Plane

Use the Distance Formula and the Pythagorean Theorem to find the distance, to the nearest tenth, from A to B.



Method 1

Use the Distance Formula. Substitute the values for the coordinates of A and B into the Distance Formula.

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{[2 - (-2)]^2 + (-2 - 3)^2}$$
$$= \sqrt{4^2 + (-5)^2}$$
$$= \sqrt{16 + 25}$$
$$= \sqrt{41}$$
$$\approx 6.4$$

Method 2

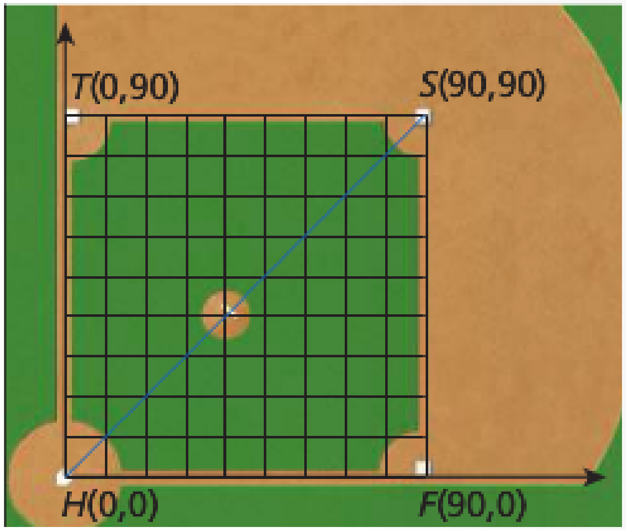
Use the Pythagorean Theorem. Count the units for sides a and b.

$a = 4$ and $b = 5$.

$$c^2 = a^2 + b^2$$
$$= 4^2 + 5^2$$
$$= 16 + 25$$
$$= 41$$
$$c = \sqrt{41}$$
$$c \approx 6.4$$

EXAMPLE 5 Sports Application

The four bases on a baseball field form a square with 90 ft sides. When a player throws the ball from home plate to second base, what is the distance of the throw, to the nearest tenth?



Set up the field on a coordinate plane so that home plate H is at the origin, first base F has coordinates $(90, 0)$, second base S has coordinates $(90, 90)$, and third base T has coordinates $(0, 90)$.

The distance HS from home plate to second base is the length of the hypotenuse of a right triangle.

$$HS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(90 - 0)^2 + (90 - 0)^2}$$

$$= \sqrt{90^2 + 90^2}$$

$$= \sqrt{8100 + 8100}$$

$$= \sqrt{16{,}200}$$

$$\approx 127.3 \text{ ft}$$

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Can you exchange the coordinates (x_1, y_1) and (x_2, y_2) in the Midpoint Formula and still find the correct midpoint? Explain.
2. A right triangle has sides lengths of r, s, and t. Given that $s^2 + t^2 = r^2$, which variables represent the lengths of the legs and which variable represents the length of the hypotenuse?
3. Do you always get the same result using the Distance Formula to find distance as you do when using the Pythagorean Theorem? Explain your answer.
4. Why do you think that most cities are laid out in a rectangular grid instead of a triangular or circular grid?
5. **GET ORGANIZED** Copy and complete the graphic organizer below. In each box, write a formula. Then make a sketch that will illustrate the formula.

Know it! Note