

COMMON
CORE

2-1

Using Inductive Reasoning to Make Conjectures

Prep for CC.9-12.G.CO.9 Prove theorems about lines and angles. Also Prep for CC.9-12.G.CO.10, Prep for CC.9-12.G.CO.11, Prep for CC.9-12.G.SRT.4

Objectives

Use inductive reasoning to identify patterns and make conjectures.

Find counterexamples to disprove conjectures.

Vocabulary

inductive reasoning
conjecture
counterexample

Who uses this?

Biologists use inductive reasoning to develop theories about migration patterns.

Biologists studying the migration patterns of California gray whales developed two theories about the whales' route across Monterey Bay. The whales either swam directly across the bay or followed the shoreline.



EXAMPLE**1****Identifying a Pattern**

Find the next item in each pattern.

A Monday, Wednesday, Friday, ...

Alternating days of the week make up the pattern.

The next day is Sunday.

B 3, 6, 9, 12, 15, ...

Multiples of 3 make up the pattern. The next multiple is 18.

C ←, ↘, ↑, ...

In this pattern, the figure rotates 45° clockwise each time.

The next figure is ↗.



1. Find the next item in the pattern 0.4, 0.04, 0.004, ...

When several examples form a pattern and you assume the pattern will continue, you are applying *inductive reasoning*. **Inductive reasoning** is the process of reasoning that a rule or statement is true because specific cases are true. You may use inductive reasoning to draw a conclusion from a pattern. A statement you believe to be true based on inductive reasoning is called a **conjecture**.

EXAMPLE 2 Making a Conjecture

Complete each conjecture.

A The product of an even number and an odd number is ? .

List some examples and look for a pattern.

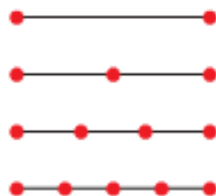
$$(2)(3) = 6 \qquad (2)(5) = 10 \qquad (4)(3) = 12 \qquad (4)(5) = 20$$

The product of an even number and an odd number is even.

Complete each conjecture.

B The number of segments formed by n collinear points is ? .

Draw a segment. Mark points on the segment, and count the number of individual segments formed. Be sure to include overlapping segments.



Points	Segments
2	1
3	$2 + 1 = 3$
4	$3 + 2 + 1 = 6$
5	$4 + 3 + 2 + 1 = 10$

The number of segments formed by n collinear points is the sum of the whole numbers less than n .

EXAMPLE**3*****Biology Application***

To learn about the migration behavior of California gray whales, biologists observed whales along two routes. For seven days they counted the numbers of whales seen along each route. Make a conjecture based on the data.

Numbers of Whales Each Day							
Direct Route	1	3	0	2	1	1	0
Shore Route	7	9	5	8	8	6	7

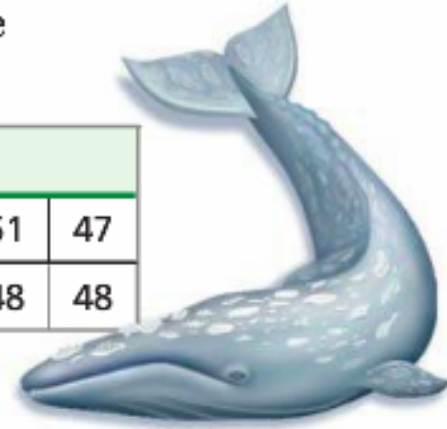
More whales were seen along the shore route each day. The data supports the conjecture that most California gray whales migrate along the shoreline.





3. Make a conjecture about the lengths of male and female whales based on the data.

Average Whale Lengths						
Length of Female (ft)	49	51	50	48	51	47
Length of Male (ft)	47	45	44	46	48	48



To show that a conjecture is always true, you must prove it.
To show that a conjecture is false, you have to find only one example in which the conjecture is not true. This case is called a **counterexample**.
A counterexample can be a drawing, a statement, or a number.



Inductive Reasoning

1. Look for a pattern
2. Make a conjecture.
3. Prove the conjecture or find a counterexample.

EXAMPLE**4****Finding a Counterexample**

Show that each conjecture is false by finding a counterexample.

A For all positive numbers n , $\frac{1}{n} \leq n$.

Pick positive values for n and substitute them into the equation to see if the conjecture holds.

Let $n = 1$. Since $\frac{1}{n} = 1$ and $1 \leq 1$, the conjecture holds.

Let $n = 2$. Since $\frac{1}{n} = \frac{1}{2}$ and $\frac{1}{2} \leq 2$, the conjecture holds.

Let $n = \frac{1}{2}$. Since $\frac{1}{n} = \frac{1}{\frac{1}{2}} = 2$ and $2 \not\leq \frac{1}{2}$, the conjecture is false.

$n = \frac{1}{2}$ is a counterexample.

B For any three points in a plane, there are three different lines that contain two of the points.



Draw three collinear points.

If the three points are collinear, the conjecture is false.

C The temperature in Abilene, Texas, never exceeds 100°F during the spring months (March, April, and May).

Monthly High Temperatures ($^\circ\text{F}$) in Abilene, Texas											
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
88	89	97	99	107	109	110	107	106	103	92	89

The temperature in May was 107°F , so the conjecture is false.



Show that each conjecture is false by finding a counterexample.

4a. For any real number x , $x^2 \geq x$.

4b. Supplementary angles are adjacent.

4c. The radius of every planet in the solar system is less than 50,000 km.



Planets' Diameters (km)

Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
4880	12,100	12,800	6790	143,000	121,000	51,100	49,500



THINK AND DISCUSS

1. Can you prove a conjecture by giving one example in which the conjecture is true? Explain your reasoning.

2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, describe the steps of the inductive reasoning process.

