COMMON CORE

2-2 Conditional Statements

Prep for CC.9-12.G.CO.9 Prove theorems about lines and angles. *Also* **Prep for CC.9-12.G.CO.10, Prep for CC.9-12.G.CO.11, Prep for CC.9-12.G.SRT.4**

Objectives

Identify, write, and analyze the truth value of conditional statements.

Write the inverse, converse, and contrapositive of a conditional statement.

Vocabulary

conditional statement
hypothesis
conclusion
truth value
negation
converse
inverse
contrapositive
logically equivalent statements

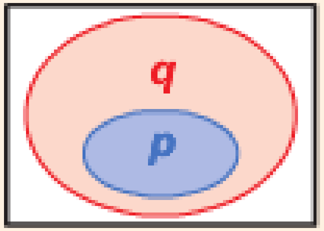
Why learn this?

To identify a species of butterfly, you must know what characteristics one butterfly species has that another does not.

It is thought that the viceroy butterfly mimics the bad-tasting monarch butterfly to avoid being eaten by birds. By comparing the appearance of the two butterfly species, you can make the following conjecture:

If a butterfly has a curved black line on its hind wing, then it is a viceroy.

Conditional Statements

DEFINITION	SYMBOLS	VENN DIAGRAM
A **conditional statement** is a statement that can be written in the form "if p, then q."	$p \rightarrow q$	
The **hypothesis** is the part p of a conditional statement following the word *if*.		
The **conclusion** is the part q of a conditional statement following the word *then*.		

By phrasing a conjecture as an if-then statement, you can quickly identify its hypothesis and conclusion.

EXAMPLE 1

Identifying the Parts of a Conditional Statement

Writing Math

"If *p*, then *q*" can also be written as "if *p*, *q*," "*q*, if *p*," "*p* implies *q*," and "*p* only if *q*."

Identify the hypothesis and conclusion of each conditional.

A **If a butterfly has a curved black line on its hind wing, then it is a viceroy.**

Hypothesis: A butterfly has a curved black line on its hind wing.

Conclusion: The butterfly is a Viceroy.

B **A number is an integer if it is a natural number.**

Hypothesis: A number is a natural number.

Conclusion: The number is an integer.

CHECK IT OUT!

1. Identify the hypothesis and conclusion of the statement "A number is divisible by 3 if it is divisible by 6."

Many sentences without the words *if* and *then* can be written as conditionals. To do so, identify the sentence's hypothesis and conclusion by figuring out which part of the statement depends on the other.

EXAMPLE 2 Writing a Conditional Statement

Write a conditional statement from each of the following.

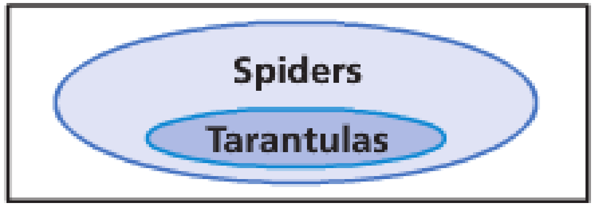
A **The midpoint M of a segment bisects the segment.**

The midpoint M of a segment bisects the segment.

Identify the hypothesis and conclusion.

Conditional: If M is the midpoint of a segment, then M bisects the segment.

B



The inner oval represents the hypothesis, and the outer oval represents the conclusion.

Conditional: If an animal is a tarantula, then it is a spider.

CHECK IT OUT!

2. Write a conditional statement from the sentence "Two angles that are complementary are acute."

A conditional statement has a **truth value** of either true (T) or false (F). It is false only when the hypothesis is true and the conclusion is false. Consider the conditional "If I get paid, I will take you to the movie." If I don't get paid, I haven't broken my promise. So the statement is still true.

To show that a conditional statement is false, you need to find only one counterexample where the hypothesis is true and the conclusion is false.

EXAMPLE 3 Analyzing the Truth Value of a Conditional Statement

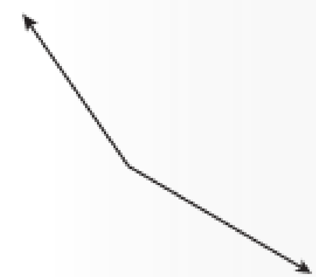
Determine if each conditional is true. If false, give a counterexample.

A **If today is Sunday, then tomorrow is Monday.**

When the hypothesis is true, the conclusion is also true because Monday follows Sunday. So the conditional is true.

B **If an angle is obtuse, then it has a measure of 100°.**

You can draw an obtuse angle whose measure is not 100°. In this case, the hypothesis is true, but the conclusion is false. Since you can find a counterexample, the conditional is false.



C **If an odd number is divisible by 2, then 8 is a perfect square.**

An odd number is never divisible by 2, so the hypothesis is false. The number 8 is not a perfect square, so the conclusion is false. However, the conditional is true because the hypothesis is false.

Remember!

If the hypothesis is false, the conditional statement is true, regardless of the truth value of the conclusion.

CHECK IT OUT!

3. Determine if the conditional "If a number is odd, then it is divisible by 3" is true. If false, give a counterexample.

The **negation** of statement p is "not p," written as $\sim p$. The negation of the statement "M is the midpoint of $\overline{AB}$" is "M is *not* the midpoint of $\overline{AB}$." The negation of a true statement is false, and the negation of a false statement is true. Negations are used to write related conditional statements.

Know it! Note

Related Conditionals

DEFINITION	SYMBOLS
A conditional is a statement that can be written in the form "If p, then q."	$p \to q$
The **converse** is the statement formed by exchanging the hypothesis and conclusion.	$q \to p$
The **inverse** is the statement formed by negating the hypothesis and the conclusion.	$\sim p \to \sim q$
The **contrapositive** is the statement formed by both exchanging and negating the hypothesis and conclusion.	$\sim q \to \sim p$

EXAMPLE 4 Biology Application


Moth

Write the converse, inverse, and contrapositive of the conditional statement. Use the photos to find the truth value of each.


Butterfly

If an insect is a butterfly, then it has four wings.

If **an insect is a butterfly,** then **it has four wings.**

Converse: If **an insect has four wings,** then **it is a butterfly.**

A moth also is an insect with four wings. So the converse is false.

Inverse: If **an insect is not a butterfly,** then **it does not have four wings.**

A moth is not a butterfly, but it has four wings. So the inverse is false.

Contrapositive: If **an insect does not have four wings,** then **it is not a butterfly.**

Butterflies must have four wings. So the contrapositive is true.

CHECK IT OUT!

4. Write the converse, inverse, and contrapositive of the conditional statement "If an animal is a cat, then it has four paws." Find the truth value of each.

Helpful Hint

The logical equivalence of a conditional and its contrapositive is known as the Law of Contrapositive.

In the example above, the conditional statement and its contrapositive are both true, and the converse and inverse are both false. Related conditional statements that have the same truth value are called **logically equivalent statements**. A conditional and its contrapositive are logically equivalent, and so are the converse and inverse.

Statement	Example	Truth Value
Conditional	If $m\angle A = 95°$, then $\angle A$ is obtuse.	T
Converse	If $\angle A$ is obtuse, then $m\angle A = 95°$.	F
Inverse	If $m\angle A \neq 95°$, then $\angle A$ is not obtuse.	F
Contrapositive	If $\angle A$ is not obtuse, then $m\angle A \neq 95°$.	T

However, the converse of a true conditional is not necessarily false. All four related conditionals can be true, or all four can be false, depending on the statement.

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. If a conditional statement is false, what are the truth values of its hypothesis and conclusion?
2. What is the truth value of a conditional whose hypothesis is false?
3. Can a conditional statement and its converse be logically equivalent? Support your answer with an example.
4. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write the definition and give an example.

Know it! Note