

COMMON  
CORE

2-3

# Using Deductive Reasoning to Verify Conjectures

Prep for CC.9-12.G.CO.9 Prove theorems about lines and angles. Also Prep for CC.9-12.G.CO.10, Prep for CC.9-12.G.CO.11, Prep for CC.9-12.G.SRT.4

## Objective

Apply the Law of Detachment and the Law of Syllogism in logical reasoning.

## Vocabulary

deductive reasoning

## Why learn this?

You can use inductive and deductive reasoning to decide whether a common myth is accurate.

You have learned that one counterexample is enough to disprove a conjecture. But to prove that a conjecture is true, you must use *deductive reasoning*. **Deductive reasoning** is the process of using logic to draw conclusions from given facts, definitions, and properties.

**Media Application**

Urban legends and modern myths spread quickly through the media. Many Web sites and television shows are dedicated to confirming or disproving such myths. Is each conclusion a result of inductive or deductive reasoning?

- A** There is a myth that toilets and sinks drain in opposite directions in the Southern and Northern Hemispheres. However, if you were to observe sinks draining in the two hemispheres, you would see that this myth is false. Since the conclusion is based on a pattern of observation, it is a result of inductive reasoning.
- B** There is a myth that you should not touch a baby bird that has fallen from its nest because the mother bird will disown the baby if she detects human scent. However, biologists have shown that birds cannot detect human scent. Therefore, the myth cannot be true. The conclusion is based on logical reasoning from scientific research. It is a result of deductive reasoning.





1. There is a myth that an eelskin wallet will demagnetize credit cards because the skin of the electric eels used to make the wallet holds an electric charge. However, eelskin products are not made from electric eels. Therefore, the myth cannot be true. Is this conclusion a result of inductive or deductive reasoning?

In deductive reasoning, if the given facts are true and you apply the correct logic, then the conclusion must be true. The Law of Detachment is one valid form of deductive reasoning.

A blue banner with the text 'Know it!' in white, next to a yellow notepad with the word 'Note' written in blue cursive.

**Know it!**

### **Law of Detachment**

If  $p \rightarrow q$  is a true statement and  $p$  is true, then  $q$  is true.

**EXAMPLE****2****Verifying Conjectures by Using the Law of Detachment**

Determine if each conjecture is valid by the Law of Detachment.

- A** Given: If two segments are congruent, then they have the same length.

$$\overline{AB} \cong \overline{XY}.$$

Conjecture:  $AB = XY$

Identify the **hypothesis** and **conclusion** in the given conditional.

If **two segments are congruent**, then **they have the same length**.

The given statement  $\overline{AB} \cong \overline{XY}$  matches the hypothesis of a true conditional. By the Law of Detachment  $AB = XY$ . The conjecture is valid.

- B** Given: If you are tardy 3 times, you must go to detention.

Shea is in detention.

Conjecture: Shea was tardy at least 3 times.

Identify the **hypothesis** and **conclusion** in the given conditional.

If **you are tardy 3 times**, **you must go to detention**.

The given statement "**Shea is in detention**" matches the conclusion of a true conditional. But this does not mean the hypothesis is true. Shea could be in detention for another reason. The conjecture is not valid.



2. Determine if the conjecture is valid by the Law of Detachment.  
Given: If a student passes his classes, the student is eligible to play sports. Ramon passed his classes.  
Conjecture: Ramon is eligible to play sports.

Another valid form of deductive reasoning is the Law of Syllogism. It allows you to draw conclusions from two conditional statements when the conclusion of one is the hypothesis of the other.

A graphic element consisting of a yellow sticky note with a blue arrow pointing to the right. The text "Know it!" is written in red and blue on the note, and "Note" is written in blue cursive below it.

**Know it!**

*Note*

### **Law of Syllogism**

If  $p \rightarrow q$  and  $q \rightarrow r$  are true statements, then  $p \rightarrow r$  is a true statement.

**EXAMPLE****3****Verifying Conjectures by Using the Law of Syllogism**

Determine if each conjecture is valid by the Law of Syllogism.

**A** Given: If  $m\angle A < 90^\circ$ , then  $\angle A$  is acute. If  $\angle A$  is acute, then it is not a right angle.

Conjecture: If  $m\angle A < 90^\circ$ , then it is not a right angle.

Let  $p$ ,  $q$ , and  $r$  represent the following.

$p$ : The measure of an angle is less than  $90^\circ$ .

$q$ : The angle is acute.

$r$ : The angle is not a right angle.

You are given that  $p \rightarrow q$  and  $q \rightarrow r$ . Since  $q$  is the conclusion of the first conditional and the hypothesis of the second conditional, you can conclude that  $p \rightarrow r$ . The conjecture is valid by the Law of Syllogism.

Determine if each conjecture is valid by the Law of Syllogism.

**B** Given: If a number is divisible by 4, then it is divisible by 2.  
If a number is even, then it is divisible by 2.

Conjecture: If a number is divisible by 4, then it is even.

Let  $x$ ,  $y$ , and  $z$  represent the following.

$x$ : A number is divisible by 4.

$y$ : A number is divisible by 2.

$z$ : A number is even.

You are given that  $x \rightarrow y$  and  $z \rightarrow y$ . The Law of Syllogism cannot be used to draw a conclusion since  $y$  is the conclusion of both conditionals. Even though the conjecture  $x \rightarrow z$  is true, the logic used to draw the conclusion is not valid.

**EXAMPLE****4****Applying the Laws of Deductive Reasoning**

Draw a conclusion from the given information.

**A** Given: If a team wins 10 games, then they play in the finals. If a team plays in the finals, then they travel to Boston. The Ravens won 10 games.

Conclusion: The Ravens will travel to Boston.

**B** Given: If two angles form a linear pair, then they are adjacent. If two angles are adjacent, then they share a side.  $\angle 1$  and  $\angle 2$  form a linear pair.

Conclusion:  $\angle 1$  and  $\angle 2$  share a side.



4. Draw a conclusion from the given information.  
Given: If a polygon is a triangle, then it has three sides.  
If a polygon has three sides, then it is not a quadrilateral. Polygon  $P$  is a triangle.

## THINK AND DISCUSS

1. Could “A square has exactly two sides” be the conclusion of a valid argument? If so, what do you know about the truth value of the given information?
2. Explain why writing conditional statements as symbols might help you evaluate the validity of an argument.
3. **GET ORGANIZED** Copy and complete the graphic organizer. Write each law in your own words and give an example of each.

