

# Biconditional Statements and Definitions

Prep for CC.9-12.G.CO.9 Prove theorems about lines and angles. Also Prep for CC.9-12.G.CO.10, Prep for CC.9-12.G.CO.11, Prep for CC.9-12.G.SRT.4

## Objective

Write and analyze biconditional statements.

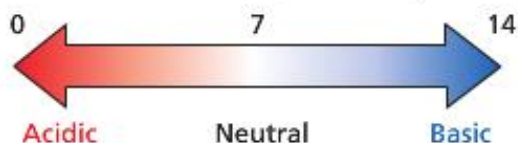
## Vocabulary

biconditional statement  
definition  
polygon  
triangle  
quadrilateral

## Who uses this?

A gardener can plan the color of the hydrangeas she plants by checking the pH of the soil.

The pH of a solution is a measure of the concentration of hydronium ions in the solution. If a solution has a pH less than 7, it is an acid. Also, if a solution is an acid, it has a pH less than 7.



## Writing Math

The biconditional “ $p$  if and only if  $q$ ” can also be written as “ $p$  iff  $q$ ” or  $p \leftrightarrow q$ .

When you combine a conditional statement and its converse, you create a *biconditional statement*. A **biconditional statement** is a statement that can be written in the form “ $p$  if and only if  $q$ .” This means “if  $p$ , then  $q$ ” and “if  $q$ , then  $p$ .”

$$p \leftrightarrow q \text{ means } p \rightarrow q \text{ and } q \rightarrow p$$

So you can define an acid with the following biconditional statement: A solution is an acid if and only if it has a pH less than 7.

**EXAMPLE****1****Identifying the Conditionals within a Biconditional Statement**

Write the conditional statement and converse within each biconditional.

- A** Two angles are congruent if and only if their measures are equal.

Let  $p$  and  $q$  represent the following.

$p$ : Two angles are congruent.

$q$ : Two angle measures are equal.

The two parts of the biconditional  $p \leftrightarrow q$  are  $p \rightarrow q$  and  $q \rightarrow p$ .

Conditional: If two angles are congruent, then their measures are equal.

Converse: If two angle measures are equal, then the angles are congruent.

- B** A solution is a base  $\leftrightarrow$  it has a pH greater than 7.

Let  $x$  and  $y$  represent the following.

$x$ : A solution is a base.

$y$ : A solution has a pH greater than 7.

The two parts of the biconditional  $x \leftrightarrow y$  are  $x \rightarrow y$  and  $y \rightarrow x$ .

Conditional: If a solution is a base, then it has a pH greater than 7.

Converse: If a solution has a pH greater than 7, then it is a base.



Write the conditional statement and converse within each biconditional.

**1a.** An angle is acute iff its measure is greater than  $0^\circ$  and less than  $90^\circ$ .

**1b.** Cho is a member if and only if he has paid the \$5 dues.

**EXAMPLE****2****Writing a Biconditional Statement**

For each conditional, write the converse and a biconditional statement.

**A** If  $2x + 5 = 11$ , then  $x = 3$ .

Converse: If  $x = 3$ , then  $2x + 5 = 11$ .

Biconditional:  $2x + 5 = 11$  if and only if  $x = 3$ .

**B** If a point is a midpoint, then it divides the segment into two congruent segments.

Converse: If a point divides a segment into two congruent segments, then the point is a midpoint.

Biconditional: A point is a midpoint if and only if it divides the segment into two congruent segments.



For each conditional, write the converse and a biconditional statement.

**2a.** If the date is July 4th, then it is Independence Day.

**2b.** If points lie on the same line, then they are collinear.

For a biconditional statement to be true, both the conditional statement and its converse must be true. If either the conditional or the converse is false, then the biconditional statement is false.

**EXAMPLE****3****Analyzing the Truth Value of a Biconditional Statement**

Determine if each biconditional is true. If false, give a counterexample.

**A** A square has a side length of 5 if and only if it has an area of 25.

Conditional: If a square has a side length of 5, then it has an area of 25. *The conditional is true.*

Converse: If a square has an area of 25, then it has a side length of 5. *The converse is true.*

Since the conditional and its converse are true, the biconditional is true.

**B** The number  $n$  is a positive integer  $\leftrightarrow 2n$  is a natural number.

Conditional: If  $n$  is a positive integer, then  $2n$  is a natural number. *The conditional is true.*

Converse: If  $2n$  is a natural number, then  $n$  is a positive integer. *The converse is false.*

If  $2n = 1$ , then  $n = \frac{1}{2}$ , which is not an integer. Because the converse is false, the biconditional is false.




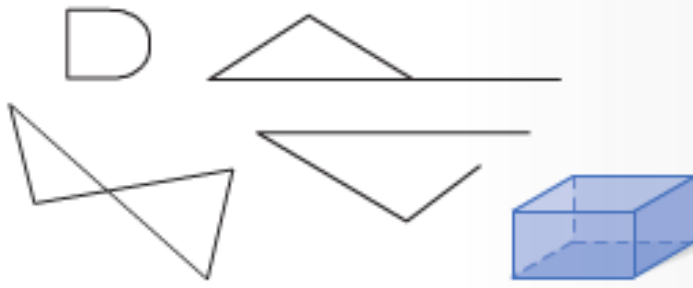
Determine if each biconditional is true. If false, give a counterexample.

**3a.** An angle is a right angle iff its measure is  $90^\circ$ .

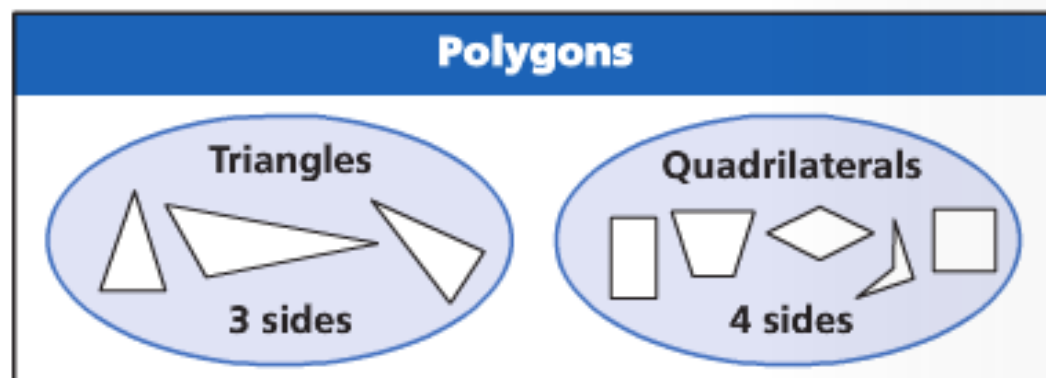
**3b.**  $y = -5 \leftrightarrow y^2 = 25$

In geometry, biconditional statements are used to write *definitions*. A **definition** is a statement that describes a mathematical object and can be written as a true biconditional. Most definitions in the glossary are not written as biconditional statements, but they can be. The “if and only if” is implied.

In the glossary, a **polygon** is defined as a closed plane figure formed by three or more line segments. Each segment intersects exactly two other segments only at their endpoints, and no two segments with a common endpoint are collinear.

Polygons	Not Polygons
	

A **triangle** is defined as a three-sided polygon, and a **quadrilateral** is a four-sided polygon.



A good, precise definition can be used forward and backward. For example, if a figure is a quadrilateral, then it is a four-sided polygon. If a figure is a four-sided polygon, then it is a quadrilateral. To make sure a definition is precise, it helps to write it as a biconditional statement.

### EXAMPLE

4

### Writing Definitions as Biconditional Statements

#### Helpful Hint

Think of definitions as being reversible. Postulates, however, are not necessarily true when reversed.

Write each definition as a biconditional.

**A** A triangle is a three-sided polygon.

A figure is a triangle if and only if it is a three-sided polygon.

**B** A segment bisector is a ray, segment, or line that divides a segment into two congruent segments.

A ray, segment, or line is a segment bisector if and only if it divides a segment into two congruent segments.



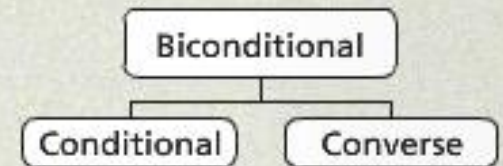
Write each definition as a biconditional.

**4a.** A quadrilateral is a four-sided polygon.

**4b.** The measure of a straight angle is  $180^\circ$ .

## THINK AND DISCUSS

1. How do you determine if a biconditional statement is true or false?
2. Compare a triangle and a quadrilateral.
3. **GET ORGANIZED** Copy and complete the graphic organizer. Use the definition of a polygon to write a conditional, converse, and biconditional in the appropriate boxes.



Know it!

Note