

COMMON
CORE

2-5

Algebraic Proof

Prep for CC.9-12.G.CO.9 Prove theorems about lines and angles. Also Prep for CC.9-12.G.CO.10, Prep for CC.9-12.G.CO.11, Prep for CC.9-12.G.SRT.4

Objectives

Review properties of equality and use them to write algebraic proofs.

Identify properties of equality and congruence.

Vocabulary

proof

Who uses this?

Game designers and animators solve equations to simulate motion. (See Example 2.)

A **proof** is an argument that uses logic, definitions, properties, and previously proven statements to show that a conclusion is true.

If you've ever solved an equation in Algebra, then you've already done a proof! An algebraic proof uses algebraic properties such as the properties of equality and the Distributive Property.



Remember!

The Distributive Property states that

$$a(b + c) = ab + ac.$$

Properties of Equality

Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $ac = bc$.
Division Property of Equality	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Reflexive Property of Equality	$a = a$
Symmetric Property of Equality	If $a = b$, then $b = a$.
Transitive Property of Equality	If $a = b$ and $b = c$, then $a = c$.
Substitution Property of Equality	If $a = b$, then b can be substituted for a in any expression.

As you have learned, if you start with a true statement and each logical step is valid, then your conclusion is valid.

An important part of writing a proof is giving justifications to show that every step is valid. For each justification, you can use a definition, postulate, property, or a piece of information that is given.

EXAMPLE**1****Solving an Equation in Algebra**

Solve the equation $-5 = 3n + 1$. Write a justification for each step.

$$-5 = 3n + 1$$

Given equation

$$\underline{-1} \quad \underline{-1}$$

Subtraction Property of Equality

$$-6 = 3n$$

Simplify.

$$\frac{-6}{3} = \frac{3n}{3}$$

Division Property of Equality

$$-2 = n$$

Simplify.

$$n = -2$$

Symmetric Property of Equality



1. Solve the equation $\frac{1}{2}t = -7$. Write a justification for each step.

EXAMPLE 2

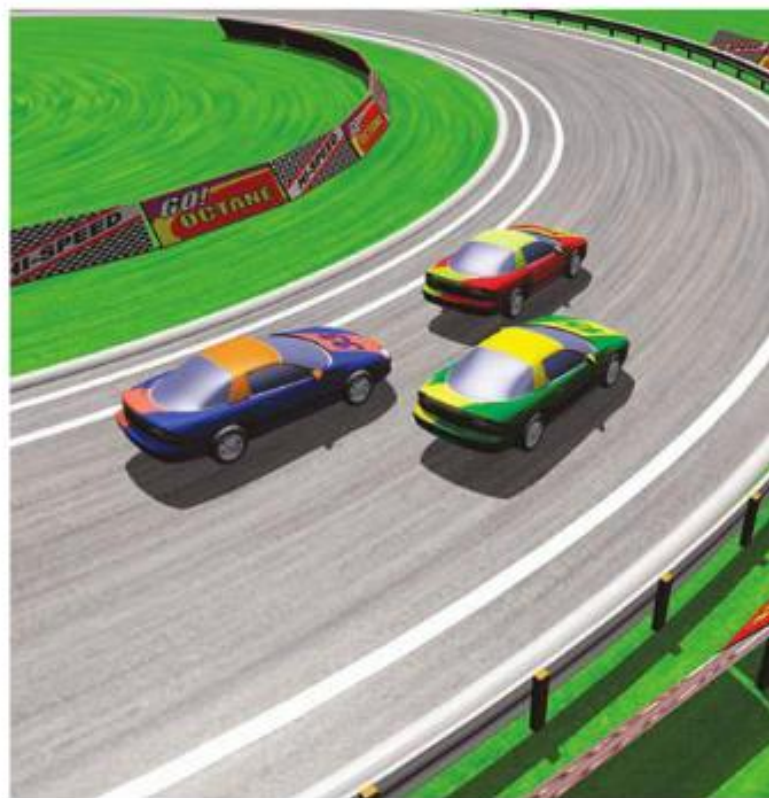
Problem-Solving Application

To simulate the motion of an object in a computer game, the designer uses the formula $sr = 3.6p$ to find the number of pixels the object must travel during each second of animation. In the formula, s is the desired speed of the object in kilometers per hour, r is the scale of pixels per meter, and p is the number of pixels traveled per second.

The graphics in a game are based on a scale of 6 pixels per meter. The designer wants to simulate a vehicle moving at 75 km/h. How many pixels must the vehicle travel each second? Solve the equation for p and justify each step.



Make sense of problems and persevere in solving them.



1 Understand the Problem

The answer will be the number of pixels traveled per second.

List the important information:

- $sr = 3.6p$
- $s = 75 \text{ km/h}$
- p : pixels traveled per second
- $r = 6 \text{ pixels per meter}$

2 Make a Plan

Substitute the given information into the formula and solve.

3 Solve

$$sr = 3.6p$$

Given equation

$$(75)(6) = 3.6p$$

Substitution Property of Equality

$$450 = 3.6p$$

Simplify.

$$\frac{450}{3.6} = \frac{3.6p}{3.6}$$

Division Property of Equality

$$125 = p$$

Simplify.

$$p = 125 \text{ pixels}$$

Symmetric Property of Equality

4 Look Back

Check your answer by substituting it back into the original formula.

$$sr = 3.6p$$

$$(75)(6) = 3.6(125)$$

$$450 = 450 \quad \checkmark$$

EXAMPLE**3****Solving an Equation in Geometry**

Write a justification for each step.

$$KM = KL + LM$$

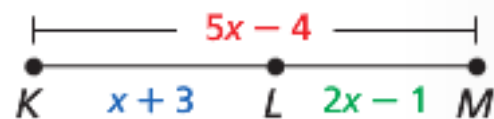
$$5x - 4 = (x + 3) + (2x - 1)$$

$$5x - 4 = 3x + 2$$

$$2x - 4 = 2$$

$$2x = 6$$

$$x = 3$$



Segment Addition Postulate

Substitution Property of Equality

Simplify.

Subtraction Property of Equality

Addition Property of Equality

Division Property of Equality



3. Write a justification for each step.

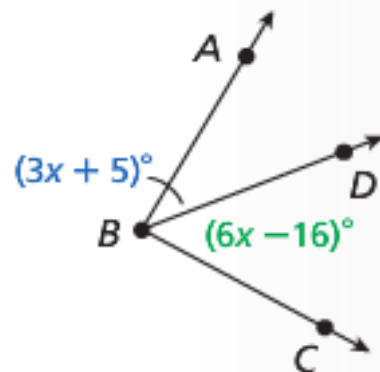
$$m\angle ABC = m\angle ABD + m\angle DBC$$

$$8x^\circ = (3x + 5)^\circ + (6x - 16)^\circ$$

$$8x = 9x - 11$$

$$-x = -11$$

$$x = 11$$



$$m\angle ABC = 8x^\circ$$

You have learned that segments with equal lengths are congruent and angles with equal measures are congruent. So the Reflexive, Symmetric, and Transitive Properties of Equality have corresponding properties of congruence.

Know it!

Note

Properties of Congruence

SYMBOLS	EXAMPLE
<p>Reflexive Property of Congruence figure $A \cong$ figure A (Reflex. Prop. of \cong)</p>	$\overline{EF} \cong \overline{EF}$
<p>Symmetric Property of Congruence If figure $A \cong$ figure B, then figure $B \cong$ figure A. (Sym. Prop. of \cong)</p>	If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$.
<p>Transitive Property of Congruence If figure $A \cong$ figure B and figure $B \cong$ figure C, then figure $A \cong$ figure C. (Trans. Prop. of \cong)</p>	If $\overline{PQ} \cong \overline{RS}$ and $\overline{RS} \cong \overline{TU}$, then $\overline{PQ} \cong \overline{TU}$.

EXAMPLE**4****Identifying Properties of Equality and Congruence****Remember!**

Numbers are equal ($=$) and figures are congruent (\cong).

Identify the property that justifies each statement.

- | | | |
|----------|--|--------------------------|
| A | $m\angle 1 = m\angle 1$ | Reflex. Prop. of $=$ |
| B | $\overline{XY} \cong \overline{VW}$, so $\overline{VW} \cong \overline{XY}$. | Sym. Prop. of \cong |
| C | $\angle ABC \cong \angle ABC$ | Reflex. Prop. of \cong |
| D | $\angle 1 \cong \angle 2$, and $\angle 2 \cong \angle 3$. So $\angle 1 \cong \angle 3$. | Trans. Prop. of \cong |



Identify the property that justifies each statement.

- | | |
|---|---|
| 4a. $DE = GH$, so $GH = DE$. | 4b. $94^\circ = 94^\circ$ |
| 4c. $0 = a$, and $a = x$. So $0 = x$. | 4d. $\angle A \cong \angle Y$, so $\angle Y \cong \angle A$. |

THINK AND DISCUSS

1. Tell what property you would use to solve the equation $\frac{k}{6} = 3.5$.
2. Explain when to use a congruence symbol instead of an equal sign.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of the property, using the correct symbol.

Property	Equality	Congruence
Reflexive		
Symmetric		
Transitive		

Know it!

Note