

# Geometric Proof

CC.9-12.G.CO.9 Prove geometric theorems about lines and angles.

## Objectives

Write two-column proofs.

Prove geometric theorems by using deductive reasoning.

## Vocabulary

theorem  
two-column proof

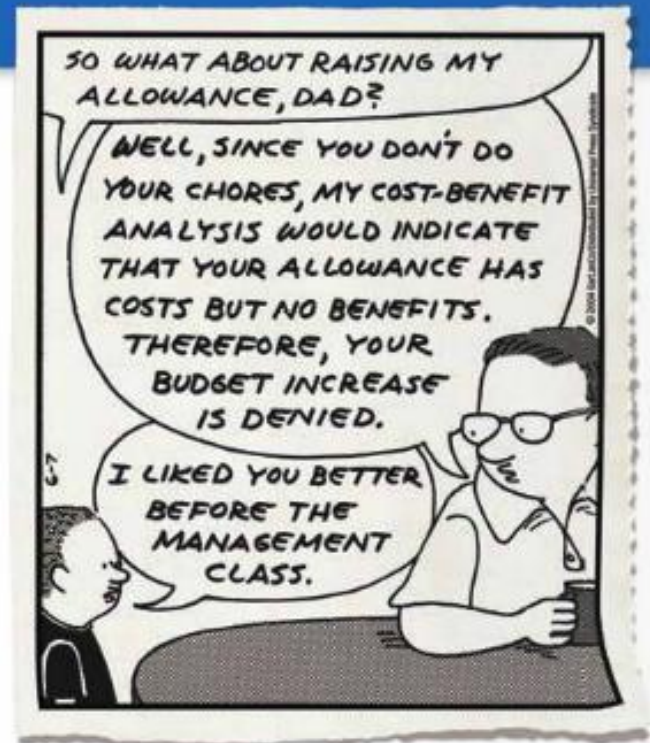
## Who uses this?

To persuade your parents to increase your allowance, your argument must be presented logically and precisely.

When writing a geometric proof, you use deductive reasoning to create a chain of logical steps that move from the hypothesis to the conclusion of the conjecture you are proving. By proving that the conclusion is true, you have proven that the original conjecture is true.



When writing a proof, it is important to justify each logical step with a reason. You can use symbols and abbreviations, but they must be clear enough so that anyone who reads your proof will understand them.



## EXAMPLE 1 Writing Justifications

### Helpful Hint

When a justification is based on more than the previous step, you can note this after the reason, as in Example 1 Step 5.

Write a justification for each step, given that  $\angle A$  and  $\angle B$  are complementary and  $\angle A \cong \angle C$ .

1.  $\angle A$  and  $\angle B$  are complementary.
2.  $m\angle A + m\angle B = 90^\circ$
3.  $\angle A \cong \angle C$
4.  $m\angle A = m\angle C$
5.  $m\angle C + m\angle B = 90^\circ$
6.  $\angle C$  and  $\angle B$  are complementary.

Given information

Def. of comp.  $\triangle$

Given information

Def. of  $\cong \triangle$

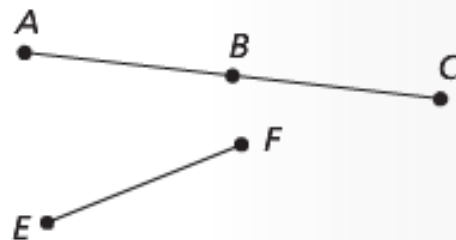
Subst. Prop. of = *Steps 2, 4*

Def. of comp.  $\triangle$



1. Write a justification for each step, given that  $B$  is the midpoint of  $\overline{AC}$  and  $\overline{AB} \cong \overline{EF}$ .

1.  $B$  is the midpoint of  $\overline{AC}$ .
2.  $\overline{AB} \cong \overline{BC}$
3.  $\overline{AB} \cong \overline{EF}$
4.  $\overline{BC} \cong \overline{EF}$



A **theorem** is any statement that you can prove. Once you have proven a theorem, you can use it as a reason in later proofs.

**Know it!**

*Note*

### Theorem

THEOREM	HYPOTHESIS	CONCLUSION
<b>2-6-1 Linear Pair Theorem</b> If two angles form a linear pair, then they are supplementary.	$\angle A$ and $\angle B$ form a linear pair.	$\angle A$ and $\angle B$ are supplementary.

**Know it!**

*Note*

### Theorem

THEOREM	HYPOTHESIS	CONCLUSION
<b>2-6-2 Congruent Supplements Theorem</b> If two angles are supplementary to the same angle (or to two congruent angles), then the two angles are congruent.	$\angle 1$ and $\angle 2$ are supplementary. $\angle 2$ and $\angle 3$ are supplementary.	$\angle 1 \cong \angle 3$

A geometric proof begins with *Given* and *Prove* statements, which restate the hypothesis and conclusion of the conjecture. In a **two-column proof**, you list the steps of the proof in the left column. You write the matching reason for each step in the right column.

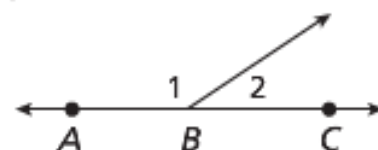
## EXAMPLE 2 Completing a Two-Column Proof

Fill in the blanks to complete a two-column proof of the Linear Pair Theorem.

Given:  $\angle 1$  and  $\angle 2$  form a linear pair.

Prove:  $\angle 1$  and  $\angle 2$  are supplementary.

Proof:



### Writing Math

Since there is no other substitution property, the Substitution Property of Equality is often written as “Substitution” or “Subst.”

Statements	Reasons
1. $\angle 1$ and $\angle 2$ form a linear pair.	1. Given
2. $\overrightarrow{BA}$ and $\overrightarrow{BC}$ form a line.	2. Def. of lin. pair
3. $m\angle ABC = 180^\circ$	3. Def. of straight $\angle$
4. a. _____?	4. $\angle$ Add. Post.
5. b. _____?	5. Subst. <i>Steps 3, 4</i>
6. $\angle 1$ and $\angle 2$ are supplementary.	6. c. _____?

Use the existing statements and reasons in the proof to fill in the blanks.

- a.  $m\angle 1 + m\angle 2 = m\angle ABC$      *The  $\angle$  Add. Post. is given as the reason.*
- b.  $m\angle 1 + m\angle 2 = 180^\circ$      *Substitute  $180^\circ$  for  $m\angle ABC$ .*
- c. Def. of supp.  $\triangle$      *The measures of supp.  $\triangle$  add to  $180^\circ$  by def.*

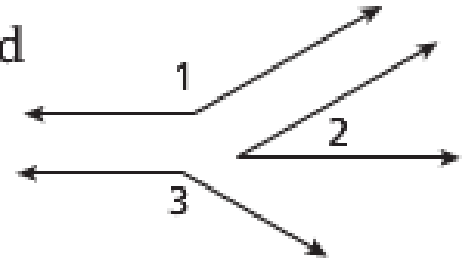


2. Fill in the blanks to complete a two-column proof of one case of the Congruent Supplements Theorem.

**Given:**  $\angle 1$  and  $\angle 2$  are supplementary, and  
 $\angle 2$  and  $\angle 3$  are supplementary.

**Prove:**  $\angle 1 \cong \angle 3$

**Proof:**



Statements	Reasons
1. a. _____?	1. Given
2. $m\angle 1 + m\angle 2 = 180^\circ$ $m\angle 2 + m\angle 3 = 180^\circ$	2. Def. of supp. $\sphericalangle$
3. b. _____?	3. Subst.
4. $m\angle 2 = m\angle 2$	4. Reflex. Prop. of =
5. $m\angle 1 = m\angle 3$	5. c. _____?
6. d. _____?	6. Def. of $\cong \sphericalangle$

Before you start writing a proof, you should plan out your logic. Sometimes you will be given a plan for a more challenging proof. This plan will detail the major steps of the proof for you.

**Know it!**  
*Note*

### Theorems

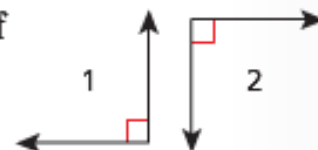
THEOREM	HYPOTHESIS	CONCLUSION
<b>2-6-3 Right Angle Congruence Theorem</b> All right angles are congruent.	$\angle A$ and $\angle B$ are right angles.	$\angle A \cong \angle B$
<b>2-6-4 Congruent Complements Theorem</b> If two angles are complementary to the same angle (or to two congruent angles), then the two angles are congruent.	$\angle 1$ and $\angle 2$ are complementary. $\angle 2$ and $\angle 3$ are complementary.	$\angle 1 \cong \angle 3$

**EXAMPLE****3****Writing a Two-Column Proof from a Plan**

Use the given plan to write a two-column proof of the Right Angle Congruence Theorem.

**Given:**  $\angle 1$  and  $\angle 2$  are right angles.

**Prove:**  $\angle 1 \cong \angle 2$



**Plan:** Use the definition of a right angle to write the measure of each angle.

Then use the Transitive Property and the definition of congruent angles.

**Proof:**

Statements	Reasons
1. $\angle 1$ and $\angle 2$ are right angles.	1. Given
2. $m\angle 1 = 90^\circ$ , $m\angle 2 = 90^\circ$	2. Def. of rt. $\angle$
3. $m\angle 1 = m\angle 2$	3. Trans. Prop. of =
4. $\angle 1 \cong \angle 2$	4. Def. of $\cong \angle$

**Helpful Hint**

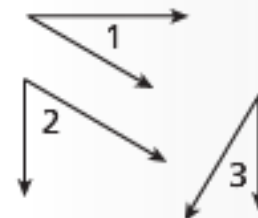
If a diagram for a proof is not provided, draw your own and mark the given information on it. But do not mark the information in the Prove statement on it.



3. Use the given plan to write a two-column proof of one case of the Congruent Complements Theorem.

**Given:**  $\angle 1$  and  $\angle 2$  are complementary,  
and  $\angle 2$  and  $\angle 3$  are complementary.

**Prove:**  $\angle 1 \cong \angle 3$



**Plan:** The measures of complementary angles add to  $90^\circ$  by definition. Use substitution to show that the sums of both pairs are equal. Use the Subtraction Property and the definition of congruent angles to conclude that  $\angle 1 \cong \angle 3$ .



### The Proof Process

1. Write the conjecture to be proven.
2. Draw a diagram to represent the hypothesis of the conjecture.
3. State the given information and mark it on the diagram.
4. State the conclusion of the conjecture in terms of the diagram.
5. Plan your argument and prove the conjecture.



## THINK AND DISCUSS

1. Which step in a proof should match the Prove statement?
2. Why is it important to include every logical step in a proof?
3. List four things you can use to justify a step in a proof.
4. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, describe the steps of the proof process.

