COMMON CORE

2-7 Flowchart and Paragraph Proofs

CC.9-12.G.CO.9 Prove theorems about lines and angles. *Also* **CC.9-12.G.CO.10**

Objectives

Write flowchart and paragraph proofs.

Prove geometric theorems by using deductive reasoning.

Vocabulary

flowchart proof

paragraph proof

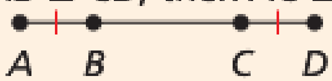
Why learn this?

Flowcharts make it easy to see how the steps of a process are linked together.

A second style of proof is a **flowchart proof**, which uses boxes and arrows to show the structure of the proof. The steps in a flowchart proof move from left to right or from top to bottom, shown by the arrows connecting each box. The justification for each step is written below the box.

Know it! Note

Theorem 2-7-1 Common Segments Theorem

THEOREM	HYPOTHESIS	CONCLUSION
Given collinear points *A*, *B*, *C*, and *D* arranged as shown, if $\overline{AB} \cong \overline{CD}$, then $\overline{AC} \cong \overline{BD}$. 	$\overline{AB} \cong \overline{CD}$	$\overline{AC} \cong \overline{BD}$

EXAMPLE 1

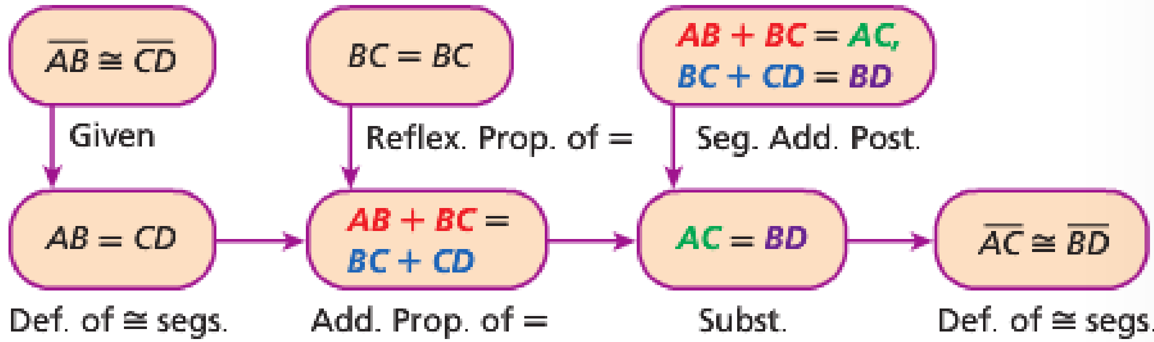
Reading a Flowchart Proof

Use the given flowchart proof to write a two-column proof of the Common Segments Theorem.

Given: $\overline{AB} \cong \overline{CD}$

Prove: $\overline{AC} \cong \overline{BD}$

Flowchart proof:



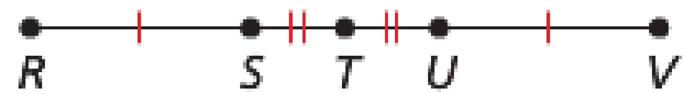
Two-column proof:

Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$	**1.** Given
2. $AB = CD$	**2.** Def. of ≅ segs.
3. $BC = BC$	**3.** Reflex. Prop. of =
4. $AB + BC = BC + CD$	**4.** Add. Prop. of =
5. $AB + BC = AC$, $BC + CD = BD$	**5.** Seg. Add. Post.
6. $AC = BD$	**6.** Subst.
7. $\overline{AC} \cong \overline{BD}$	**7.** Def. of ≅ segs.

CHECK IT OUT!

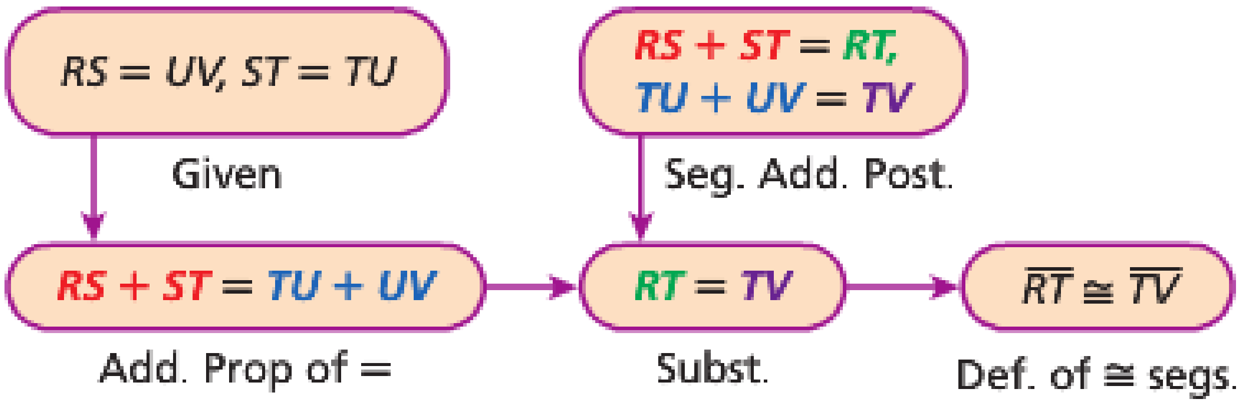
1. Use the given flowchart proof to write a two-column proof.

Given: $RS = UV$, $ST = TU$



Prove: $\overline{RT} \cong \overline{TV}$

Flowchart proof:



EXAMPLE 2

Writing a Flowchart Proof

Use the given two-column proof to write a flowchart proof of the Converse of the Common Segments Theorem.

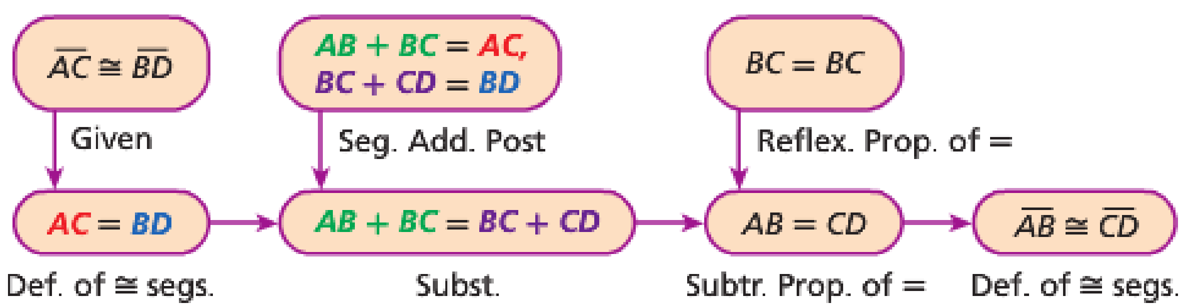
Given: $\overline{AC} \cong \overline{BD}$

Prove: $\overline{AB} \cong \overline{CD}$

Two-column proof:

Statements	Reasons
1. $\overline{AC} \cong \overline{BD}$	1. Given
2. $AC = BD$	2. Def. of ≅ segs.
3. $AB + BC = AC$, $BC + CD = BD$	3. Seg. Add. Post.
4. $AB + BC = BC + CD$	4. Subst. *Steps 2, 3*
5. $BC = BC$	5. Reflex. Prop. of =
6. $AB = CD$	6. Subtr. Prop. of =
7. $\overline{AB} \cong \overline{CD}$	7. Def. of ≅ segs.

Flowchart proof:



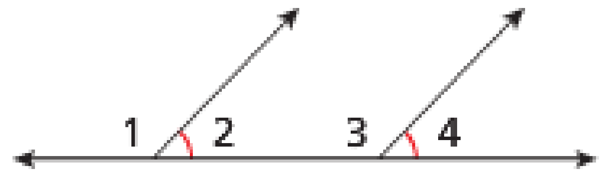
Helpful Hint

Like the converse of a conditional statement, the converse of a theorem is found by switching the hypothesis and conclusion.

2. Use the given two-column proof to write a flowchart proof.

Given: $\angle 2 \cong \angle 4$

Prove: $m\angle 1 = m\angle 3$



Two-column proof:

Statements	Reasons
1. $\angle 2 \cong \angle 4$	1. Given
2. $\angle 1$ and $\angle 2$ are supplementary. $\angle 3$ and $\angle 4$ are supplementary.	2. Lin. Pair Thm.
3. $\angle 1 \cong \angle 3$	3. $\cong$ Supps. Thm.
4. $m\angle 1 = m\angle 3$	4. Def. of $\cong$ $\measuredangle$s

A **paragraph proof** is a style of proof that presents the steps of the proof and their matching reasons as sentences in a paragraph. Although this style of proof is less formal than a two-column proof, you still must include every step.

Know it! Note

Theorems

THEOREM		HYPOTHESIS	CONCLUSION
2-7-2	**Vertical Angles Theorem** Vertical angles are congruent.	$\angle A$ and $\angle B$ are vertical angles.	$\angle A \cong \angle B$
2-7-3	If two congruent angles are supplementary, then each angle is a right angle. ($\cong$ $\angle$s supp. $\rightarrow$ rt. $\angle$s)	$\angle 1 \cong \angle 2$ $\angle 1$ and $\angle 2$ are supplementary.	$\angle 1$ and $\angle 2$ are right angles.

EXAMPLE 3 **Reading a Paragraph Proof**

Use the given paragraph proof to write a two-column proof of the Vertical Angles Theorem.

Given: ∠1 and ∠3 are vertical angles.

Prove: ∠1 ≅ ∠3

Paragraph proof: ∠1 and ∠3 are vertical angles, so they are formed by intersecting lines. Therefore ∠1 and ∠2 are a linear pair, and ∠2 and ∠3 are a linear pair. By the Linear Pair Theorem, ∠1 and ∠2 are supplementary, and ∠2 and ∠3 are supplementary. So by the Congruent Supplements Theorem, ∠1 ≅ ∠3.

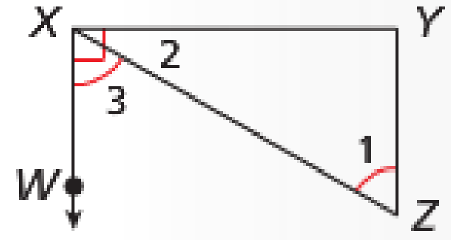
Two-column proof:

Statements	Reasons
1. ∠1 and ∠3 are vertical angles.	**1.** Given
2. ∠1 and ∠3 are formed by intersecting lines.	**2.** Def. of vert. ∠s
3. ∠1 and ∠2 are a linear pair. ∠2 and ∠3 are a linear pair.	**3.** Def. of lin. pair
4. ∠1 and ∠2 are supplementary. ∠2 and ∠3 are supplementary.	**4.** Lin. Pair Thm.
5. ∠1 ≅ ∠3	**5.** ≅ Supps. Thm.

3. Use the given paragraph proof to write a two-column proof.

Given: $\angle WXY$ is a right angle. $\angle 1 \cong \angle 3$

Prove: $\angle 1$ and $\angle 2$ are complementary.

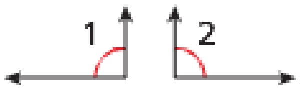


Paragraph proof: Since $\angle WXY$ is a right angle, $m\angle WXY = 90°$ by the definition of a right angle. By the Angle Addition Postulate, $m\angle WXY = m\angle 2 + m\angle 3$. By substitution, $m\angle 2 + m\angle 3 = 90°$. Since $\angle 1 \cong \angle 3$, $m\angle 1 = m\angle 3$ by the definition of congruent angles. Using substitution, $m\angle 2 + m\angle 1 = 90°$. Thus by the definition of complementary angles, $\angle 1$ and $\angle 2$ are complementary.

EXAMPLE 4 Writing a Paragraph Proof

Use the given two-column proof to write a paragraph proof of Theorem 2-7-3.

Given: ∠1 and ∠2 are supplementary. ∠1 ≅ ∠2
Prove: ∠1 and ∠2 are right angles.



Two-column proof:

Statements	Reasons
1. ∠1 and ∠2 are supplementary. ∠1 ≅ ∠2	1. Given
2. m∠1 + m∠2 = 180°	2. Def. of supp. ∡
3. m∠1 = m∠2	3. Def. of ≅ ∡ *Step 1*
4. m∠1 + m∠1 = 180°	4. Subst. *Steps 2, 3*
5. 2m∠1 = 180°	5. Simplification
6. m∠1 = 90°	6. Div. Prop. of =
7. m∠2 = 90°	7. Trans. Prop. of = *Steps 3, 6*
8. ∠1 and ∠2 are right angles.	8. Def. of rt. ∠

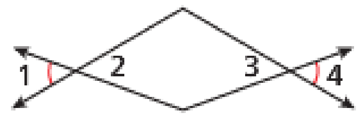
Paragraph proof: ∠1 and ∠2 are supplementary, so m∠1 + m∠2 = 180° by the definition of supplementary angles. They are also congruent, so their measures are equal by the definition of congruent angles. By substitution, m∠1 + m∠1 = 180°, so m∠1 = 90° by the Division Property of Equality. Because m∠1 = m∠2, m∠2 = 90° by the Transitive Property of Equality. So both are right angles by the definition of a right angle.

CHECK IT OUT!

4. Use the given two-column proof to write a paragraph proof.

Given: $\angle 1 \cong \angle 4$

Prove: $\angle 2 \cong \angle 3$



Two-column proof:

Statements	Reasons
1. $\angle 1 \cong \angle 4$	**1.** Given
2. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	**2.** Vert. ∡ Thm.
3. $\angle 2 \cong \angle 4$	**3.** Trans. Prop. of $\cong$ *Steps 1, 2*
4. $\angle 2 \cong \angle 3$	**4.** Trans. Prop. of $\cong$ *Steps 2, 3*

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Explain why there might be more than one correct way to write a proof.

2. Describe the steps you take when writing a proof.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, describe the proof style in your own words.