Matrix Review Answers

$$A=\begin{bmatrix}2&3&-2\\1&4&5\\0&1&7\end{bmatrix},\quad B=\begin{bmatrix}-3&-2&1\\0&5&5\\1&2&-1\end{bmatrix},\quad C=\begin{bmatrix}8&0&1\\-6&4&3\end{bmatrix}$$

Perform the indicated matrix operations.

1) -A $\begin{bmatrix}-2&-3&2\\-1&-4&-5\\0&-1&-7\end{bmatrix}$

2) 2A - B $\begin{bmatrix}7&8&-5\\2&3&5\\-1&0&15\end{bmatrix}$

3) B - A $\begin{bmatrix}-5&-5&3\\-1&1&0\\1&1&-8\end{bmatrix}$

Solve for 'x' and 'y'.

4) $\begin{bmatrix}x+3&-3\\2&3y+1\end{bmatrix}=\begin{bmatrix}10&-3\\2&10\end{bmatrix}$

$x+3=10$
$x=7$

$3y+1=10$
$3y=9$
$y=3$

5) $\begin{bmatrix}-4&21\\-4y-3&1\end{bmatrix}=\begin{bmatrix}-4&-2x+5\\-19&1\end{bmatrix}$

$21=-2x+5$
$-2x=16$
$x=-8$

$-4y-3=-19$
$-4y=-16$
$y=4$

6) $Given: A=\begin{bmatrix}1&-1&2&-2\\1&-1&3&-3\\1&-2&3&-2\end{bmatrix},$ Identify a_{23}

$a_{23}=3$

Find each product if it exists:

7) $\begin{bmatrix}-6&2\\4&0\end{bmatrix}\begin{bmatrix}3&2\\1&-1\end{bmatrix}$

$\begin{bmatrix}-18+2&-12-2\\12+0&8+0\end{bmatrix}=\begin{bmatrix}-16&-14\\12&8\end{bmatrix}$

8) $\begin{bmatrix}-3&-3&7\end{bmatrix}\begin{bmatrix}9&1\\0&3\\3&5\end{bmatrix}$

$\begin{bmatrix}-27+0+21&-3-9+35\end{bmatrix}=\begin{bmatrix}-6&23\end{bmatrix}$

9) $\begin{bmatrix}1&-1&0\\2&0&-3\end{bmatrix}\begin{bmatrix}1&-1&0\\2&0&-3\end{bmatrix}$

Not Happy----Can't be done.

10) Find the Determinant for [A].

$$A = \begin{bmatrix} 4 & 2 \\ 6 & 3 \end{bmatrix}$$

12 12

12 - 12 = 0

11) Find the Determinant for [B].

0 -4 0 0 1 -18

$$B = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 0 & 1 \\ -2 & 1 & -3 \end{bmatrix} \begin{matrix} 1 & 2 \\ 3 & 0 \\ -2 & 1 \end{matrix}$$

-4 - -17 = 13

12) Find the Determinant in terms of 'x' i.e. D_x.

$$\left[\begin{array}{ccc|c} -1 & -2 & 0 & -4 \\ 1 & 2 & 1 & 7 \\ 3 & 6 & 3 & 21 \end{array}\right]$$

-24 -42 0 0 -24 -42

$$\begin{matrix} -4 & -2 & 0 & -4 & -2 \\ 7 & 2 & 1 & 7 & 2 \\ 21 & 6 & 3 & 21 & 6 \end{matrix}$$

-66 - -66 = 0

13) Find the inverse matrix A.

$$\begin{bmatrix} 1 & 5 & -3 \\ 2 & 1 & -1 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 3 & -4 & 5 \\ 5 & -7 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A x B = I

$$B = A^{-1} = \begin{bmatrix} 1 & -1 & 2 \\ 3 & -4 & 5 \\ 5 & -7 & 9 \end{bmatrix}$$

14) Find a, b, c and d. Write them in matrix form.

$$\begin{bmatrix} 5 & 8 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5a + 8c = 1 & 5b + 8d = 0 \\ 2a + 3c = 0 & 2b + 3d = 1 \end{bmatrix}$$

$a = -3$

$c = 2$

$b = 8$

$d = -5$

$$\begin{bmatrix} -3 & 8 \\ 2 & -5 \end{bmatrix}$$

15) Find the inverse.

$$\begin{bmatrix} 3 & 8 \\ 4 & 12 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{matrix} 3a + 8c = 1 & 3b + 8d = 0 \\ 4a + 12c = 0 & 4b + 12d = 1 \end{matrix}$$

$c = -1$

$a = 3$

$d = \frac{3}{4}$

$b = -2$

$$A^{-1} = \begin{bmatrix} 3 & -2 \\ -1 & \frac{3}{4} \end{bmatrix}$$

16) If [A] is a coefficient matrix, [B] is the inverse of [A] and [C] is the constant matrix, then find the solution matrix.

$$A = \begin{bmatrix} 2 & 4 \\ 0 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{2} & \frac{2}{3} \\ 0 & -\frac{1}{3} \end{bmatrix}, \quad C = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2} & \frac{2}{3} \\ 0 & -\frac{1}{3} \end{bmatrix} x \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 2+2 \\ 0-1 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

17) Determine whether matrix [B] is the inverse of [A]. (SHOW YOUR WORK).

$$A = \begin{bmatrix} -3 & -2 & 1 \\ 0 & 5 & 5 \\ 1 & 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} -\frac{1}{2} & 0 & -\frac{1}{2} \\ \frac{1}{6} & \frac{1}{15} & \frac{1}{2} \\ -\frac{1}{6} & \frac{2}{15} & -\frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} \frac{3}{2} - \frac{1}{3} - \frac{1}{6} & 0 - \frac{2}{15} + \frac{2}{15} & \frac{3}{2} - 1 - \frac{1}{2} \\ 0 + \frac{5}{6} - \frac{5}{6} & 0 + \frac{1}{3} + \frac{10}{15} & 0 + \frac{5}{2} - \frac{5}{2} \\ -\frac{1}{2} + \frac{1}{3} + \frac{1}{6} & 0 + \frac{2}{15} - \frac{2}{15} & -\frac{1}{2} + 1 + \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad YES$$

18) Use the inverse matrix to solve this system.

$$x + 2y = 16$$
$$2x + y = 11$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a+2c=1 & b+2d=0 \\ 2a+c=0 & 2b+d=1 \end{bmatrix}$$

$$a = -\frac{1}{3} \quad d = -\frac{1}{3}$$
$$c = \frac{2}{3} \quad b = \frac{2}{3}$$

$$\begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} 16 \\ 11 \end{bmatrix} = \begin{bmatrix} -\frac{16}{3} + \frac{22}{3} \\ \frac{32}{3} - \frac{11}{3} \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$$

19) Use Cramer's rule of discriminates and diagonals to solve the system:

$$x + y + z = 6$$
$$x + y - z = 0$$
$$3x + 2y + z = 10$$

1 -3 2 3 -2 1

$$D = \begin{array}{ccccc} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 1 & 1 \\ 3 & 2 & 1 & 3 & 2 \end{array}$$

0 - 2 = -2

6 -10 0 10 -12 0

$$D_x = \begin{array}{ccccc} 6 & 1 & 1 & 6 & 1 \\ 0 & 1 & -1 & 0 & 1 \\ 10 & 2 & 1 & 10 & 2 \end{array}$$

-4 - -2 = -2

0 -18 10 0 -10 6

$$D_y = \begin{array}{ccccc} 1 & 6 & 1 & 1 & 6 \\ 1 & 0 & -1 & 1 & 0 \\ 3 & 10 & 1 & 3 & 10 \end{array}$$

-8 - -4 = -4

10 0 12 18 0 10

$$D_z = \begin{array}{ccccc} 1 & 1 & 6 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 3 & 2 & 10 & 3 & 2 \end{array}$$

22 - 28 = -6

$$x = \frac{D_x}{D} = \frac{-2}{-2} = 1; \qquad y = \frac{D_y}{D} = \frac{-4}{-2} = 2; \qquad z = \frac{D_z}{D} = \frac{-6}{-2} = 3$$