

: Trigonometric Identities

<b>Quotient Identities</b>		<b>Reciprocal Identities</b>		
$\tan \theta = \frac{\sin \theta}{\cos \theta}$	$\cot \theta = \frac{\cos \theta}{\sin \theta}$	$\csc \theta = \frac{1}{\sin \theta}$	$\sec \theta = \frac{1}{\cos \theta}$	$\cot \theta = \frac{1}{\tan \theta}$

<b>Pythagorean Identities</b>		
$\sin^2 \theta + \cos^2 \theta = 1$	$\tan^2 \theta + 1 = \sec^2 \theta$	$1 + \cot^2 \theta = \csc^2 \theta$

<b>Even-Odd Identities</b>		
$\sin(-\theta) = -\sin \theta$	$\cos(-\theta) = \cos \theta$	$\tan(-\theta) = -\tan \theta$
$\csc(-\theta) = -\csc \theta$	$\sec(-\theta) = \sec \theta$	$\cot(-\theta) = -\cot \theta$

Examples: Establish the identity (AKA Prove the identity!)

1. $\frac{\sin^2(-\theta) - \cos^2(-\theta)}{\sin(-\theta) - \cos(-\theta)} = \cos \theta - \sin \theta$	reasons	2. $\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = 2 \csc \theta$	reasons
1. $\frac{\sin^2(-\theta) - \cos^2(-\theta)}{\sin(-\theta) - \cos(-\theta)} = \cos \theta - \sin \theta$	given	$\frac{\sin \theta}{1 + \cos \theta} \left( \frac{\sin \theta}{\sin \theta} \right) + \frac{1 + \cos \theta}{\sin \theta} \left( \frac{1 + \cos \theta}{1 + \cos \theta} \right)$	given
2. $\frac{\sin(-\theta)^2 - \cos(-\theta)^2}{\sin(-\theta) - \cos(-\theta)}$	def. of f(x) □	$\frac{\sin^2 \theta + 1 + 2 \cos \theta + \cos^2 \theta}{(\sin \theta)(1 + \cos \theta)}$	mult. ident.
3. $\frac{-(\sin \theta)^2 - \cos^2 \theta}{-\sin \theta - \cos \theta}$	odd/even ident.	$\frac{\sin^2 \theta + \cos^2 \theta + 1 + 2 \cos \theta}{(\sin \theta)(1 + \cos \theta)}$	distribute
4. $\frac{\sin^2 \theta - \cos^2 \theta}{-\sin \theta - \cos \theta}$	multiplication	$\frac{1 + 1 + 2 \cos \theta}{(\sin \theta)(1 + \cos \theta)}$	pyth. ident.
5. $\frac{(\sin \theta - \cos \theta)(\sin \theta + \cos \theta)}{-1(\sin \theta + \cos \theta)}$	distributive prop	$\frac{2(1 + \cos \theta)}{(\sin \theta)(1 + \cos \theta)}$	distributive prop.
3. $\frac{\tan \theta + \cot \theta}{\sec \theta \csc \theta} = 1$	simplify	$\frac{2(1 + \cos \theta)}{(\sin \theta)(1 + \cos \theta)}$	mult. ident.
6. $\frac{\sin \theta - \cos \theta}{-1}$	distribute -1	$2 \csc \theta$	recip. ident.

reason	given	quotient ident	reciprocal ident.	mult. ident.	distribute	pythag. ident	def. of divide by 1
	$\frac{1 + \tan \theta + \cot \theta}{\sec \theta \csc \theta} = 1$	$\frac{\sin \theta + \cos \theta}{\cos \theta \sin \theta} \cdot \sec \theta \csc \theta$	$\frac{\sin \theta + \cos \theta}{\cos \theta \sin \theta} \cdot \frac{1}{\cos \theta \sin \theta}$	$\frac{\sin \theta + \cos \theta}{\cos \theta \sin \theta} \cdot \frac{1}{\cos \theta \sin \theta} \cdot \frac{\sin \theta \cos \theta}{\sin \theta \cos \theta}$	$\frac{\sin^2 \theta + \cos^2 \theta}{1}$	$\frac{\sin^2 \theta + \cos^2 \theta = 1}{1}$	$\frac{1}{1} = 1$