

$$\begin{aligned} \textcircled{1} \quad \cos \theta \sec \theta &= \boxed{1} \\ \downarrow \\ \cos \theta \left( \frac{1}{\cos \theta} \right) \\ &= \boxed{1} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad \tan \theta \cot \theta &= \boxed{1} \\ \downarrow \\ \cancel{\tan \theta} \frac{1}{\cancel{\tan \theta}} \\ \downarrow \\ \boxed{1} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad \sin \theta \sec \theta &= \boxed{\tan \theta} \\ \downarrow \\ \sin \theta \frac{1}{\cos \theta} \\ \downarrow \\ \frac{\sin \theta}{\cos \theta} \\ \downarrow \\ \boxed{\tan \theta} \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad \sin \theta \cot \theta &= \boxed{\cos \theta} \\ \downarrow \\ \cancel{\sin \theta} \left( \frac{\cos \theta}{\cancel{\sin \theta}} \right) \\ \downarrow \\ \boxed{\cos \theta} \end{aligned}$$

$$\begin{aligned} \textcircled{5} \quad \frac{\csc \theta}{\sec \theta} &= \boxed{\cot \theta} \\ \downarrow \\ \frac{\frac{1}{\sin \theta}}{\frac{1}{\cos \theta}} \\ \downarrow \\ \frac{\cos \theta}{\sin \theta} \\ \downarrow \\ \boxed{\cot \theta} \end{aligned}$$

$$\begin{aligned} \textcircled{6} \quad \cot \theta \sec \theta &= \boxed{\csc \theta} \\ \downarrow \\ \left( \frac{\cos \theta}{\sin \theta} \right) \left( \frac{1}{\cos \theta} \right) \\ \downarrow \\ \frac{1}{\sin \theta} \\ \downarrow \\ \boxed{\csc \theta} \end{aligned}$$

$$\begin{aligned} \textcircled{7} \quad (1 + \cos \theta)(1 - \cos \theta) &= \boxed{\sin^2 \theta} \\ \downarrow \\ 1 - \cos^2 \theta \\ \downarrow \\ \boxed{\sin^2 \theta} \end{aligned}$$

$$\textcircled{8} \quad \cos^2 \theta (\sec^2 \theta - 1) = \boxed{\sin^2 \theta}$$

$$\downarrow$$
$$\cos^2 \theta (\tan^2 \theta)$$

$$\downarrow$$
$$\cancel{\cos^2 \theta} \left( \frac{\sin^2 \theta}{\cancel{\cos^2 \theta}} \right)$$

$$\downarrow$$
$$\boxed{\sin^2 \theta}$$

$$\textcircled{9} \quad \cos^2 \theta - \sin^2 \theta = \boxed{2 \cos^2 \theta - 1}$$

$$\downarrow$$
$$\cos^2 \theta - (1 - \cos^2 \theta)$$

$$\downarrow$$
$$\cos^2 \theta - 1 + \cos^2 \theta$$

$$\downarrow$$
$$\boxed{2 \cos^2 \theta - 1}$$

$$\textcircled{10} \quad (\tan \theta + \cot \theta) \tan \theta = \boxed{\sec^2 \theta}$$

$$\downarrow$$
$$\tan^2 \theta + \cot \theta \tan \theta$$

$$\downarrow$$
$$\tan^2 \theta + \frac{1}{\tan \theta} (\cancel{\tan \theta})$$

$$\downarrow$$
$$\tan^2 \theta + 1$$

$$\downarrow$$
$$\boxed{\sec^2 \theta}$$

$$\textcircled{11} \quad \frac{\sin \theta}{\csc \theta} + \frac{\cos \theta}{\sec \theta} = \boxed{1}$$

$$\downarrow$$
$$\frac{\sin \theta}{\frac{1}{\sin \theta}} + \frac{\cos \theta}{\frac{1}{\cos \theta}}$$

$$\downarrow$$
$$\sin^2 \theta + \cos^2 \theta$$

$$\downarrow$$
$$\boxed{1}$$

$$\textcircled{12} \quad 1 - 2 \sin^2 \theta = \boxed{2 \cos^2 \theta - 1}$$

$$\downarrow$$
$$1 - 2(1 - \cos^2 \theta)$$

$$\downarrow$$
$$1 - 2 + 2 \cos^2 \theta$$

$$\downarrow$$
$$-1 + 2 \cos^2 \theta$$

$$\downarrow$$
$$\boxed{2 \cos^2 \theta - 1}$$

$$\textcircled{13} \quad (1 + \sin \theta)(1 - \sin \theta) = \boxed{\frac{1}{\sec^2 \theta}}$$

$$\downarrow$$
$$1 - \sin^2 \theta$$

$$\downarrow$$
$$\cos^2 \theta$$

$$\downarrow$$
$$\boxed{\frac{1}{\sec^2 \theta}}$$

$$\textcircled{14} (1 - \sin^2 \theta)(1 + \tan^2 \theta) = \boxed{1}$$

$$\downarrow \qquad \qquad \downarrow$$
$$\cos^2 \theta (\sec^2 \theta)$$

$$\downarrow$$
$$\cancel{\cos^2 \theta} \left( \frac{1}{\cancel{\cos^2 \theta}} \right)$$

$$\downarrow$$
$$\boxed{1}$$

$$\textcircled{15} \sec \theta - \cos \theta = \boxed{\tan \theta \sin \theta}$$

$$\downarrow$$
$$\frac{1}{\cos \theta} - \cos \theta$$

$$\downarrow$$
$$\frac{1 - \cos^2 \theta}{\cos \theta}$$

$$\downarrow$$
$$\frac{\sin^2 \theta}{\cos \theta}$$

$$\downarrow$$
$$\frac{\sin \theta \sin \theta}{\cos \theta}$$

$$\downarrow$$
$$\boxed{\tan \theta \sin \theta}$$

$$\textcircled{16} \quad \frac{\sin \theta + \cos \theta}{\cos \theta} = \boxed{1 + \tan \theta}$$

$$\downarrow$$
$$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\cos \theta}$$

$$\downarrow$$
$$\tan \theta + 1$$

$$\downarrow$$
$$\boxed{1 + \tan \theta}$$

$$\textcircled{17} \quad \frac{\csc^2 \theta}{1 + \tan^2 \theta} = \boxed{\cot^2 \theta}$$

$$\downarrow$$
$$\frac{\csc^2 \theta}{\sec^2 \theta}$$

$$\downarrow$$
$$\frac{1}{\sin^2 \theta}$$
$$\frac{1}{\cos^2 \theta}$$

$$\downarrow$$
$$\frac{\cos^2 \theta}{\sin^2 \theta}$$

$$\downarrow$$
$$\boxed{\cot^2 \theta}$$

$$\textcircled{18} \quad \sin \theta + \cos \theta \cot \theta = \boxed{\csc \theta}$$

$$\downarrow$$
$$\sin \theta + \cos \theta \left( \frac{\cos \theta}{\sin \theta} \right)$$

$$\downarrow$$
$$\sin \theta + \frac{\cos^2 \theta}{\sin \theta}$$

$$\downarrow$$
$$\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta}$$

$$\downarrow$$
$$\frac{1}{\sin \theta}$$

$$\downarrow$$
$$\boxed{\csc \theta}$$

$$\textcircled{19} \quad \sin \theta (\csc \theta - \sin \theta) = \cos^2 \theta$$

$$\downarrow$$
$$\sin \theta \left( \frac{1}{\sin \theta} - \sin \theta \right)$$

$$\downarrow$$
$$1 - \sin^2 \theta$$

$$\downarrow$$
$$\cos^2 \theta$$

$$\textcircled{20} \quad \cot \theta + \tan \theta = \boxed{\csc \theta \sec \theta}$$

$$\begin{array}{c} \downarrow \qquad \downarrow \\ \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} \end{array}$$

$$\downarrow \\ \frac{\cos^2 \theta + \sin^2 \theta}{\cos \theta \sin \theta}$$

$$\downarrow \\ \frac{1}{\sin \theta \cos \theta}$$

$$\downarrow \\ \boxed{\csc \theta \sec \theta}$$

$$\textcircled{21} \quad \csc \theta - \sin \theta = \boxed{\cot \theta \cos \theta}$$

$$\downarrow \\ \frac{1}{\sin \theta} - \sin \theta$$

$$\downarrow \\ \frac{1 - \sin^2 \theta}{\sin \theta}$$

$$\downarrow \\ \frac{\cos^2 \theta}{\sin \theta}$$

$$\downarrow \\ \frac{\cos \theta \cos \theta}{\sin \theta}$$

$$\downarrow \\ \boxed{\cot \theta \cos \theta}$$

$$(22) \quad \cos \theta (\tan \theta + \cot \theta) = \boxed{\csc \theta}$$

$$\cos \theta \left( \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right)$$

$$\sin \theta + \frac{\cos^2 \theta}{\sin \theta}$$

$$\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta}$$

$$\frac{1}{\sin \theta}$$

$$\boxed{\csc \theta}$$

$$(23) \quad \frac{\sec^2 \theta - 1}{\sec^2 \theta} = \boxed{\sin^2 \theta}$$

$$\frac{\tan^2 \theta}{\sec^2 \theta}$$

$$\frac{\sin^2 \theta}{\cos^2 \theta}$$

$$\frac{1}{\cancel{\cos^2 \theta}}$$

$$\boxed{\sin^2 \theta}$$



$$(24) \quad (\tan \theta + \cot \theta)(\cos \theta + \sin \theta) = \boxed{\sec \theta + \csc \theta}$$

$$\begin{array}{c} \downarrow \qquad \downarrow \\ \left( \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right) (\cos \theta + \sin \theta) \end{array}$$

$$\downarrow \\ \left( \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} \right) (\cos \theta + \sin \theta)$$

$$\downarrow \\ \left( \frac{1}{\cos \theta \sin \theta} \right) (\cos \theta + \sin \theta)$$

$$\downarrow \\ \frac{\cancel{\cos \theta}}{\cancel{\cos \theta} \sin \theta} + \frac{\cancel{\sin \theta}}{\cos \theta \cancel{\sin \theta}}$$

$$\downarrow \\ \frac{1}{\sin \theta} + \frac{1}{\cos \theta}$$

$$\downarrow \\ \csc \theta + \sec \theta$$

$$\leftarrow \\ \boxed{\sec \theta + \csc \theta}$$

$$(25) (\cos^2 \theta - 1)(\tan^2 \theta + 1) = \boxed{1 - \sec^2 \theta}$$

$$(\cos^2 \theta - 1) \sec^2 \theta$$

$$\cos^2 \theta \sec^2 \theta - \sec^2 \theta$$

$$\boxed{1 - \sec^2 \theta}$$

$$(26) (\cot \theta + \csc \theta)(\tan \theta - \sin \theta) = \boxed{\sec \theta - \cos \theta}$$

$$\left(\frac{\cos \theta}{\sin \theta} + \frac{1}{\sin \theta}\right) \left(\frac{\sin \theta}{\cos \theta} - \sin \theta\right)$$

$$\left(\frac{\cos \theta + 1}{\sin \theta}\right) \left(\frac{\sin \theta - \sin \theta \cos \theta}{\cos \theta}\right)$$

$$\frac{\cos \theta \sin \theta - \sin \theta \cos^2 \theta + \sin \theta - \sin \theta \cos \theta}{\sin \theta \cos \theta}$$

$$\frac{\sin \theta (1 - \cos^2 \theta)}{\sin \theta \cos \theta}$$

$$\frac{1}{\cos \theta} - \frac{\cos^2 \theta}{\cos \theta}$$

$$\boxed{\sec \theta - \cos \theta}$$

(27)

$$\sec \theta \csc \theta + \cot \theta = \boxed{\tan \theta + 2 \cos \theta \csc \theta}$$

$$\downarrow$$

$$\frac{1}{\sin \theta \cos \theta} + \frac{\cos \theta}{\sin \theta}$$

$$\downarrow$$

$$\frac{1 + \cos^2 \theta}{\sin \theta \cos \theta}$$

$$\downarrow$$

$$\frac{\sin^2 \theta + \cos^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}$$

$$\downarrow$$

$$\frac{\sin^2 \theta + 2 \cos^2 \theta}{\sin \theta \cos \theta}$$

$$\downarrow$$

$$\frac{\cancel{\sin \theta} \theta}{\cancel{\sin \theta} \cos \theta} + \frac{2 \cos \theta}{\cancel{\sin \theta} \cos \theta}$$

$$\downarrow$$

$$\frac{\sin \theta}{\cos \theta} + \frac{2 \cos \theta}{\sin \theta}$$

$$\downarrow$$

$$\downarrow$$

$$\boxed{\tan \theta + 2 \cos \theta \csc \theta}$$

$$(28) \quad \frac{1 + \cos^2 \theta}{\sin^2 \theta} = \boxed{2 \csc^2 \theta - 1}$$

$$\downarrow$$
$$\frac{1 + 1 - \sin^2 \theta}{\sin^2 \theta}$$

$$\downarrow$$
$$\frac{2 - \sin^2 \theta}{\sin^2 \theta}$$

$$\downarrow$$
$$\frac{2}{\sin^2 \theta} - \frac{\sin^2 \theta}{\sin^2 \theta}$$

$$\downarrow$$
$$\boxed{2 \csc^2 \theta - 1}$$

$$(29) \quad \sec^2 \theta \csc^2 \theta = \boxed{\sec^2 \theta + \csc^2 \theta}$$

$$\downarrow$$
$$\left( \frac{1}{\cos^2 \theta} \right) \left( \frac{1}{\sin^2 \theta} \right)$$

$$\downarrow$$
$$\frac{1}{\cos^2 \theta \sin^2 \theta}$$

$$\downarrow$$
$$\frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta \sin^2 \theta}$$

$$\downarrow$$
$$\frac{\cancel{\cos^2 \theta}}{\cos^2 \theta \sin^2 \theta} + \frac{\cancel{\sin^2 \theta}}{\cos^2 \theta \sin^2 \theta}$$

$$\downarrow$$
$$\frac{1}{\sin^2 \theta} + \frac{1}{\cos^2 \theta}$$

$$\downarrow$$
$$\boxed{\csc^2 \theta + \sec^2 \theta}$$

$$\textcircled{30} \quad \frac{\sec \theta - \cos \theta}{\tan \theta} = \boxed{\frac{\tan \theta}{\sec \theta}}$$

$$\downarrow$$
$$\frac{\frac{1}{\cos \theta} - \cos \theta}{\frac{\sin \theta}{\cos \theta}}$$

$$\downarrow$$
$$\frac{1 - \cos^2 \theta}{\cancel{\cos \theta}}$$
$$\frac{\sin \theta}{\cos \theta}$$

$$\downarrow$$
$$\frac{1 - \cos^2 \theta}{\sin \theta}$$

$$\downarrow$$
$$\frac{\sin \theta}{\cancel{\sin \theta}}$$

$$\downarrow$$
$$\sin \theta \cdot \left( \frac{\cos \theta}{\cos \theta} \right)$$

$$\downarrow$$
$$\tan \theta \cos \theta$$
$$\downarrow$$

$$\boxed{\frac{\tan \theta}{\sec \theta}}$$

$$(31) \quad \frac{1 + \cos \theta}{\sin \theta} + \frac{\sin \theta}{1 + \cos \theta} = \boxed{2 \csc \theta}$$

$$\downarrow$$
$$\frac{(1 + \cos \theta)^2 + \sin^2 \theta}{\sin \theta (1 + \cos \theta)}$$

$$\downarrow$$
$$\frac{\sin^2 \theta + 1 + 2 \cos \theta + \cos^2 \theta}{\sin \theta (1 + \cos \theta)}$$

$$\downarrow$$
$$\frac{1 + 1 + 2 \cos \theta}{\sin \theta (1 + \cos \theta)}$$

$$\downarrow$$
$$\frac{2 \cancel{(1 + \cos \theta)}}{\sin \theta \cancel{(1 + \cos \theta)}}$$

$$\downarrow$$
$$\frac{2}{\sin \theta} = \boxed{2 \csc \theta}$$

$$(32) \quad \tan^2 \theta - \sin^2 \theta = \boxed{\tan^2 \theta \sin^2 \theta}$$

$$\downarrow$$
$$\frac{\sin^2 \theta}{\cos^2 \theta} - \sin^2 \theta$$

$$\downarrow$$
$$\frac{\sin^2 \theta - \sin^2 \theta \cos^2 \theta}{\cos^2 \theta}$$

$$\downarrow$$
$$\frac{\sin^2 \theta (1 - \cos^2 \theta)}{\cos^2 \theta}$$

$$\downarrow$$
$$\frac{\sin^2 \theta \sin^2 \theta}{\cos^2 \theta}$$

$$\downarrow$$
$$\boxed{\tan^2 \theta \sin^2 \theta}$$

$$(33) \quad \frac{1 + \tan^2 \theta}{\tan^2 \theta} = \boxed{\csc^2 \theta}$$

$$\downarrow$$
$$\frac{\sec^2 \theta}{\tan^2 \theta}$$

$$\downarrow$$
$$\frac{\frac{1}{\cos^2 \theta}}{\frac{\sin^2 \theta}{\cos^2 \theta}}$$

$$\downarrow$$
$$\frac{1}{\sin^2 \theta} = \boxed{\csc^2 \theta}$$

$$(34) \quad \frac{\sec\theta + \csc\theta}{\sec\theta - \csc\theta} = \boxed{\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}}$$

$$\downarrow$$

$$\frac{\frac{1}{\cos\theta} + \frac{1}{\sin\theta}}{\frac{1}{\cos\theta} - \frac{1}{\sin\theta}}$$

$$\downarrow$$

$$\frac{\frac{\sin\theta + \cos\theta}{\cancel{\cos\theta} \sin\theta}}{\frac{\sin\theta - \cos\theta}{\cancel{\cos\theta} \sin\theta}}$$

$$\downarrow$$

$$\boxed{\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}}$$

$$(35) \quad \frac{1 + \sin\theta}{1 - \sin\theta} - \frac{1 - \sin\theta}{1 + \sin\theta} = \boxed{4 \tan\theta \sec\theta}$$

$$\downarrow$$

$$\frac{(1 + \sin\theta)(1 + \sin\theta) - (1 - \sin\theta)(1 - \sin\theta)}{(1 - \sin\theta)(1 + \sin\theta)}$$

$$\downarrow$$

$$\frac{\cancel{1} + 2\sin\theta + \cancel{\sin^2\theta} - \cancel{1} + 2\sin\theta - \cancel{\sin^2\theta}}{1 - \sin^2\theta}$$

$$\downarrow$$

$$\frac{4 \sin\theta}{\cos^2\theta}$$

$$\downarrow$$

$$\frac{4 \sin\theta}{\cos\theta \cos\theta} = \boxed{4 \tan\theta \sec\theta}$$



$$(36) \quad \frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta} = \boxed{2\csc^2\theta}$$

$$\downarrow$$

$$\frac{1+\cancel{\cos\theta} + 1-\cancel{\cos\theta}}{(1-\cos\theta)(1+\cos\theta)}$$

$$\downarrow$$

$$\frac{2}{1-\cos^2\theta}$$

$$\downarrow$$

$$\frac{2}{\sin^2\theta} = \boxed{2\csc^2\theta}$$

$$(37) \quad \frac{1+\csc\theta}{\sec\theta} - \cot\theta = \boxed{\cos\theta}$$

$$\downarrow$$

$$\frac{1 + \frac{1}{\sin\theta}}{\frac{1}{\cos\theta}} - \frac{\cos\theta}{\sin\theta}$$

$$\downarrow$$

$$\frac{\sin\theta + 1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} \quad \begin{array}{l} x^2 - 4 \\ (x-2)(x+2) \end{array}$$

$$\downarrow$$

$$\frac{(\sin\theta + 1)\cos\theta}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$$

$$\downarrow$$

$$\frac{\sin\theta \cos\theta + \cancel{\cos\theta} - \cancel{\cos\theta}}{\sin\theta}$$

$$\downarrow$$

$$\frac{\cancel{\sin\theta} \cos\theta}{\cancel{\sin\theta}} = \boxed{\cos\theta}$$

(38)

$$\frac{\cos \theta \cot \theta}{\cot \theta - \cos \theta} = \boxed{\frac{\cot \theta + \cos \theta}{\cos \theta \cot \theta}}$$

↓

$$\frac{\cos \theta \cot \theta}{\cot \theta - \cos \theta} \cdot \frac{\cot \theta + \cos \theta}{\cot \theta + \cos \theta}$$

↓

$$\frac{\cos \theta \cot \theta (\cot \theta + \cos \theta)}{\cot^2 \theta - \cos^2 \theta}$$

↓

$$\frac{\cos \theta \cot \theta (\cot \theta + \cos \theta)}{\frac{\cos^2 \theta}{\sin^2 \theta} - \cos^2 \theta}$$

$$\frac{\cos \theta \cot \theta (\cot \theta + \cos \theta)}{\frac{\cos^2 \theta}{\sin^2 \theta} - \cos^2 \theta}$$

↓

$$\frac{\cos \theta \cot \theta (\cot \theta + \cos \theta)}{\frac{\cos^2 \theta - \cos^2 \theta \sin^2 \theta}{\sin^2 \theta}}$$

↓

$$\frac{\cos \theta \cot \theta (\cot \theta + \cos \theta)}{\frac{\cos^2 \theta (1 - \sin^2 \theta)}{\sin^2 \theta}}$$

↓

$$\frac{\cos \theta \cot \theta (\cot \theta + \cos \theta)}{\frac{\cos^2 \theta \cos^2 \theta}{\sin^2 \theta}}$$

↓

$$\frac{\cancel{\cos \theta} \cot \theta (\cot \theta + \cancel{\cos \theta})}{\cot^2 \theta \cos^2 \theta}$$

↓

$$\boxed{\frac{\cot \theta + \cos \theta}{\cos \theta \cot \theta}}$$

$$(40) \quad \frac{\cot \theta - \tan \theta}{\sin \theta + \cos \theta} = \boxed{\csc \theta - \sec \theta}$$

↓

$$\frac{\frac{\cos \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta}}{\sin \theta + \cos \theta}$$

↓

$$\frac{\frac{\cos^2 \theta - \sin^2 \theta}{\cos \theta \sin \theta}}{\sin \theta + \cos \theta}$$

↓

$$\frac{(\cos \theta - \sin \theta)(\cancel{\cos \theta + \sin \theta})}{\cos \theta \sin \theta}$$

---

$$\frac{\cancel{\sin \theta + \cos \theta}}{\sin \theta + \cos \theta}$$

↓

$$\frac{\cos \theta - \sin \theta}{\cos \theta \sin \theta}$$

↓

$$\frac{\cancel{\cos \theta}}{\cancel{\cos \theta} \sin \theta} - \frac{\cancel{\sin \theta}}{\cos \theta \cancel{\sin \theta}}$$

↓

$$\frac{1}{\sin \theta} - \frac{1}{\cos \theta} = \boxed{\csc \theta - \sec \theta}$$

$$(41) \frac{\cot \theta - 1}{1 - \tan \theta} = \boxed{\cot \theta}$$

↓

$$\frac{\frac{\cos \theta}{\sin \theta} - 1}{1 - \frac{\sin \theta}{\cos \theta}}$$

$$\frac{\cos \theta - \sin \theta}{\sin \theta}$$

↓

$$\frac{\cancel{\cos \theta} - \sin \theta}{\sin \theta}$$

$$\frac{\cancel{\cos \theta} - \sin \theta}{\cos \theta}$$

↓

$$\frac{\cos \theta}{\sin \theta} = \boxed{\cot \theta}$$

$$(42) \frac{1 + \sec \theta}{\tan \theta + \sin \theta} = \csc \theta$$

↓

$$\frac{1 + \frac{1}{\cos \theta}}{\frac{\sin \theta}{\cos \theta} + \sin \theta}$$

$$\frac{\cos \theta + 1}{\cancel{\cos \theta}}$$

↓

$$\frac{\cos \theta + 1}{\cancel{\cos \theta}}$$

$$\frac{\sin \theta + \sin \theta \cancel{\cos \theta}}{\cancel{\cos \theta}}$$

↓

$$\frac{\cancel{\cos \theta} + 1}{\sin \theta (1 + \cancel{\cos \theta})}$$

$$\frac{1}{\sin \theta} = \boxed{\csc \theta}$$

$$\textcircled{43} \quad \csc^4 \theta - \cot^4 \theta = \boxed{\cot^2 \theta + \csc^2 \theta}$$

$$\downarrow$$
$$(\csc^2 \theta - \cot^2 \theta)(\csc^2 \theta + \cot^2 \theta)$$

$$\downarrow \qquad \qquad \qquad \downarrow$$
$$(1 + \cancel{\cot^2 \theta} - \cancel{\cot^2 \theta})(\cot^2 \theta + \csc^2 \theta)$$

$$1(\cot^2 \theta + \csc^2 \theta) = \boxed{\cot^2 \theta + \csc^2 \theta}$$

$$\textcircled{44} \quad \cos^4 \theta + \sin^2 \theta = \boxed{\sin^4 \theta + \cos^2 \theta}$$

$$\downarrow$$
$$(\cos^2 \theta)(\cos^2 \theta) + \sin^2 \theta$$

$$\downarrow \qquad \qquad \downarrow$$
$$(1 - \sin^2 \theta)(1 - \sin^2 \theta) + \sin^2 \theta$$

$$1 - \cancel{\sin^2 \theta} + \sin^4 \theta + \cancel{\sin^2 \theta}$$

$$\downarrow$$
$$1 - \sin^2 \theta + \sin^4 \theta$$

$$\downarrow$$
$$\boxed{\sin^4 \theta + \cos^2 \theta}$$

$$\textcircled{45} \quad \frac{\cos \theta}{1 - \sin \theta} = \boxed{\sec \theta + \tan \theta}$$

$$\downarrow$$
$$\frac{\cos \theta}{1 - \sin \theta} \cdot \frac{1 + \sin \theta}{1 + \sin \theta}$$

$$\downarrow$$
$$\frac{\cos \theta (1 + \sin \theta)}{1 - \sin^2 \theta}$$

$$\downarrow$$
$$\frac{\cos \theta (1 + \sin \theta)}{\cancel{\cos \theta}}$$

$$\downarrow$$
$$\frac{1 + \sin \theta}{\cos \theta} = \frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} = \boxed{\sec \theta + \tan \theta}$$

$$(46) \quad \frac{1}{\csc\theta - \cot\theta} = \boxed{\csc\theta + \cot\theta}$$

$$\downarrow$$
$$\frac{1}{\csc\theta - \cot\theta} \cdot \frac{\csc\theta + \cot\theta}{\csc\theta + \cot\theta}$$

$$\downarrow$$
$$\frac{\csc\theta + \cot\theta}{\csc^2\theta - \cot^2\theta}$$

$$\downarrow$$
$$\frac{\csc\theta + \cot\theta}{1 + \cancel{\cot^2\theta} - \cancel{\cot^2\theta}}$$

$$\downarrow$$
$$\boxed{\csc\theta + \cot\theta}$$

$$(47) \quad \frac{\tan^2\theta}{\sec\theta + 1} = \boxed{\frac{1 - \cos\theta}{\cos\theta}}$$

$$\downarrow$$
$$\frac{\sec^2\theta - 1}{\sec\theta + 1}$$

$$\downarrow$$
$$\frac{(\sec\theta - 1)(\sec\theta + 1)}{\sec\theta + 1}$$

$$\downarrow$$
$$\sec\theta - 1$$

$$\downarrow$$
$$\frac{1}{\cos\theta} - 1$$

$$\downarrow$$
$$\boxed{\frac{1 - \cos\theta}{\cos\theta}}$$

$$(48) \quad \frac{\cot \theta}{\csc \theta + 1} = \boxed{\frac{\csc \theta - 1}{\cot \theta}}$$

$$\downarrow$$
$$\frac{\cot \theta}{\csc \theta + 1} \cdot \frac{\csc \theta - 1}{\csc \theta - 1}$$

$$\downarrow$$
$$\frac{\cot \theta (\csc \theta - 1)}{\csc^2 \theta - 1}$$

$$\downarrow$$
$$\frac{\cot \theta (\csc \theta - 1)}{\cot^2 \theta}$$

$$\downarrow$$
$$\boxed{\frac{\csc \theta - 1}{\cot \theta}}$$

$$(49) \quad \frac{\cot \theta - 1}{\cot \theta + 1} = \boxed{\frac{1 - \tan \theta}{1 + \tan \theta}}$$

$$\downarrow$$
$$\frac{\frac{1}{\tan \theta} - 1}{\frac{1}{\tan \theta} + 1}$$

$$\frac{1 - \tan \theta}{1 + \tan \theta}$$

$$\downarrow$$
$$\frac{1 - \tan \theta}{\tan \theta} \cdot \frac{\tan \theta}{\tan \theta}$$
$$\frac{1 - \tan \theta}{1 + \tan \theta}$$

$$\downarrow$$
$$\boxed{\frac{1 - \tan \theta}{1 + \tan \theta}}$$

$$\textcircled{50} \quad \frac{1 + \sec \theta}{\sin \theta + \tan \theta} = \boxed{\csc \theta}$$

↓

$$\frac{1 + \frac{1}{\cos \theta}}{\sin \theta + \frac{\sin \theta}{\cos \theta}}$$

↓

$$\frac{\frac{\cos \theta + 1}{\cos \theta}}{\frac{\sin \theta \cos \theta + \sin \theta}{\cos \theta}}$$

↓

$$\frac{\cos \theta + 1}{\sin \theta \cos \theta + \sin \theta}$$

↓

$$\frac{\cancel{\cos \theta + 1}}{\sin \theta (\cancel{\cos \theta + 1})}$$

↓

$$\frac{1}{\sin \theta} = \boxed{\csc \theta}$$

$$\textcircled{51} \quad \sin^4 \theta - \cos^4 \theta = \boxed{\sin^2 \theta - \cos^2 \theta}$$

$$\downarrow$$
$$(\sin^2 \theta - \cos^2 \theta)(\sin^2 \theta + \cos^2 \theta)$$

$$\downarrow$$
$$(\sin^2 \theta - \cos^2 \theta)(1) = \boxed{\sin^2 \theta - \cos^2 \theta}$$



$$\begin{aligned} \textcircled{52} \quad \sin^4 \theta + 2\sin^2 \theta \cos^2 \theta + \cos^4 \theta &= \boxed{1} \\ &\downarrow \\ (\sin^2 \theta + \cos^2 \theta)(\sin^2 \theta + \cos^2 \theta) \\ &\downarrow \qquad \downarrow \\ (1)(1) &= \boxed{1} \end{aligned}$$

$$\begin{aligned} \textcircled{53} \quad \tan^4 \theta - \sec^4 \theta &= \boxed{1 - 2\sec^2 \theta} \\ &\downarrow \\ (\tan^2 \theta - \sec^2 \theta)(\tan^2 \theta + \sec^2 \theta) \\ &\downarrow \qquad \downarrow \\ (\cancel{\sec^2 \theta} - 1 - \cancel{\sec^2 \theta})(\sec^2 \theta - 1 + \sec^2 \theta) \\ &\qquad \downarrow \\ -1(2\sec^2 \theta - 1) \\ &\downarrow \\ \boxed{1 - 2\sec^2 \theta} \end{aligned}$$

$$\begin{aligned} \textcircled{54} \quad \sec^4 \theta - \sec^2 \theta &= \boxed{\tan^4 \theta + \tan^2 \theta} \\ &\downarrow \\ (\sec^2 \theta)(\sec^2 \theta) - \sec^2 \theta \\ &\downarrow \qquad \downarrow \\ (1 + \tan^2 \theta)(1 + \tan^2 \theta) - (1 + \tan^2 \theta) \\ &\downarrow \\ \cancel{1} + \cancel{2}\tan^2 \theta + \tan^4 \theta - \cancel{1} - \cancel{\tan^2 \theta} \\ &\downarrow \\ \boxed{\tan^4 \theta + \tan^2 \theta} \end{aligned}$$

$$(55) (\sec\theta + \tan\theta)^2 = \boxed{\frac{1 + \sin\theta}{1 - \sin\theta}}$$

$$\downarrow$$

$$\left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)^2$$

$$\downarrow$$

$$\left(\frac{1 + \sin\theta}{\cos\theta}\right)^2$$

$$\downarrow$$

$$\frac{(1 + \sin\theta)(1 + \sin\theta)}{\cos^2\theta}$$

$$\downarrow$$

$$\frac{(1 + \sin\theta)(1 + \sin\theta)}{1 - \sin^2\theta}$$

$$\downarrow$$

$$\frac{\cancel{(1 + \sin\theta)}(1 + \sin\theta)}{\cancel{(1 + \sin\theta)}(1 - \sin\theta)}$$

$$\downarrow$$

$$\boxed{\frac{1 + \sin\theta}{1 - \sin\theta}}$$

$$(56) \sec^2\theta + \tan^2\theta = \boxed{(1 - \sin^4\theta)(\sec^4\theta)}$$

$$\downarrow \quad \downarrow$$

$$\frac{1}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta}$$

$$\downarrow$$

$$\frac{1 + \sin^2\theta}{\cos^4\theta}$$

$$\downarrow$$

$$\frac{(1 + \sin^2\theta)}{\cos^2\theta} \cdot \left(\frac{1 - \sin^2\theta}{1 - \sin^2\theta}\right)$$

$$\downarrow$$

$$\frac{1 - \sin^4\theta}{(\cos^2\theta)(\cos^2\theta)} = \frac{1 - \sin^4\theta}{\cos^4\theta} = \boxed{(1 - \sin^4\theta)(\sec^4\theta)}$$

$$(57) \quad (\sin^2 \theta + \cos^2 \theta)^3 = \boxed{1}$$

$$\downarrow$$
$$(1)^3 = \boxed{1}$$

$$(58) \quad \frac{\sin \theta}{1 - \cos \theta} = \boxed{\csc \theta + \cot \theta}$$

$$\downarrow$$
$$\frac{\sin \theta}{1 - \cos \theta} \cdot \frac{1 + \cos \theta}{1 + \cos \theta}$$

$$\downarrow$$
$$\frac{\sin \theta (1 + \cos \theta)}{1 - \cos^2 \theta}$$

$$\downarrow$$
$$\frac{\cancel{\sin \theta} (1 + \cos \theta)}{\cancel{\sin \theta}}$$

$$\downarrow$$
$$\frac{1 + \cos \theta}{\sin \theta}$$

$$\downarrow$$
$$\frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta} = \boxed{\csc \theta + \cot \theta}$$