P. 482 (9-39) 3rd + p. 483 (Mixed Practice)

15) $\cot\theta = -2 \quad \sec\theta < 0$

find (a) $\sin(2\theta)$ (b) $\cos(2\theta)$ (c) $\sin\frac{\theta}{2}$ (d) $\cos\frac{\theta}{2}$

If $\cot\theta = -2$ then $\frac{x}{y} = -2$ and $x = -2$ and $y = 1$ or $x = 2$ and $y = -1$

Since $\sec\theta < 0$, $\cos\theta$ must be neg.

so $x = -2$ and $y = 1$

If $x = -2$ and $y = 1$, r must be $\sqrt{5}$ (Pythag.)

That means $\cos\theta = -\frac{2}{\sqrt{5}}$ & $\sin\theta = \frac{1}{\sqrt{5}}$

Rationalize: $\cos\theta = -\frac{2\sqrt{5}}{5}$ & $\sin\theta = \frac{\sqrt{5}}{5}$

Plug those into each identity

(a) $$\sin(2\theta) = 2\sin\theta\cos\theta = 2\left(\frac{\sqrt{5}}{5}\right)\left(-\frac{2\sqrt{5}}{5}\right) = \frac{-4(5)}{25} = \boxed{-\frac{20}{25}}$$

(b) $$\cos(2\theta) = \cos^2\theta - \sin^2\theta = (\cos\theta)^2 - (\sin\theta)^2 = \left(-\frac{2\sqrt{5}}{5}\right)^2 - \left(\frac{\sqrt{5}}{5}\right)^2 = \left(\frac{20}{25}\right) - \left(\frac{5}{25}\right) = \frac{15}{25} = \boxed{3/5}$$

c) $$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} = \pm\sqrt{\frac{1 - \frac{-2\sqrt{5}}{5}}{2}} = \pm\sqrt{\frac{1}{2} - \frac{2\sqrt{5}}{10}} = \boxed{\pm\sqrt{\frac{5-2\sqrt{5}}{10}}}$$

(d) $$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} = \boxed{\pm\sqrt{\frac{5+2\sqrt{5}}{10}}}$$

24) $\sin 195°$ (Need 2 angles from the unit circle)

$$\sin(\underset{\alpha}{135°} + \underset{\beta}{60°})$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$= (\sin 135°)(\cos 60°) + (\cos 135°)(\sin 60°)$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$= \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$$

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33) $h(2\theta)$ means $\tan 2\theta$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

According to the circle, $\tan\theta = \frac{2}{a}$

use $x^2 + y^2 = 5$ to find a

$$a^2 + y^2 = 5$$
$$a^2 + 2^2 = 5$$
$$a^2 = 1$$
$$a = 1$$

So $\tan\theta = \frac{2}{1} = 2$

$$\tan(2\theta) = \frac{2(2)}{1 - (2)^2}$$

$$= \frac{4}{1-4}$$

$$= \frac{4}{-3} \text{ or } \boxed{\frac{-4}{3}}$$

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39) $\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha}$ (you could also use the other 2 identities)

According to the circle $\cos\alpha = -\frac{1}{4}$ and $\sin\alpha = b$

Solve for b using $x^2+y^2=1$ $\quad (-\frac{1}{4})^2 + b^2 = 1$

$b^2 = 1 - \frac{1}{16}$

$b^2 = -15/16$

$b = -\frac{\sqrt{15}}{4} = \sin\alpha$

SO: $\tan\frac{\alpha}{2} = \frac{1 - -\frac{1}{4}}{-\frac{\sqrt{15}}{4}}$

mult. by $\frac{4}{4}$

$= \frac{4 - 1}{-\sqrt{15}}$

$= \frac{3}{-\sqrt{15}}$

$= -\frac{3\sqrt{15}}{15}$

$= \boxed{\frac{-\sqrt{15}}{5}}$

71) $\cos\left(2\sin^{-1}\left(\frac{3}{5}\right)\right)$

Giving you this

let $\theta = \sin^{-1}\left(\frac{3}{5}\right)$

$\sin^2\theta = (\sin\theta)^2$

$$\cos(2\theta) = 1 - 2\sin^2\theta$$

$$= 1 - 2\sin^2\left(\sin^{-1}\frac{3}{5}\right)$$

$$1 - 2\left(\sin\left(\sin^{-1}\frac{3}{5}\right)\right)^2$$

$$1 - 2\left(\frac{3}{5}\right)^2$$

$$1 - 2\left(\frac{9}{25}\right)$$

$$1 - \frac{18}{25}$$

$$\frac{25}{25} - \frac{18}{25}$$

$$= \boxed{\frac{7}{25}}$$

"GOOD LUCK MY CHILDREN"